

## Structure and dynamics of disks in galaxies

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**Abstract.** One the most cited papers in astronomy is Ken Freeman’s 1970 paper on exponential disks in galaxies. In this contribution I review what has been done in this area since then and what we can infer about systematic properties of disks in galaxies from surface photometry, HI synthesis observations and measurements of stellar kinematics. Most disks have radial truncations at  $3.6 \pm 0.6$  radial scalelengths  $h_R$ . Galaxy disk thicknesses  $h_R/h_z = 7.3 \pm 2.2$  imply that these disk cannot be “maximum disks”. I briefly discuss a recent study of the “superthin” edge-on galaxy IC 5249.

### 1. Introduction

This meeting is a tribute to Ken Freeman, his research and that with his students. Ken’s most cited paper is undoubtedly “*On the disks of spiral and S0 galaxies*” (Freeman, 1970). Although it was de Vaucouleurs (e.g. 1959), who first showed observationally that the surface brightness distributions can be described by an exponential law, it was Ken in his seminal paper who collected the material available and studied the distribution of parameters and the dynamical relations. He derived the rotation curve and angular momentum distribution for an exponential disk in centrifugal equilibrium, found that 28 of the 36 galaxies have approximately the same face-on central surface brightness (known as “Freeman’s law”) and on the basis of the rotation curves of NGC300 and M33 concluded that there must be undetected matter of at least the mass of the detected galaxy.

It is of some interest to see how Ken’s 1970-paper compares in citation rate to others. For this purpose I collected such information using the NASA Astrophysics Data System (ADS). The citation scores in ADS are not complete, but certainly indicative and internally consistent. The first exercise was to draw up a list of what I feel are the most important papers related to studies of structure of galaxies. In Table 1 I give the total citation score up to the end of 2000, that for 1999+2000 (“9/0”) and the average number of citations per year. In this listing I find only a few papers that are comparable or exceed Ken’s 1970-paper in citations. These are Ed Salpeter’s 1955 paper on star formation in the Galactic disk in which he defines the “Salpeter function”, Ivan King’s 1966 paper on the dynamics of globular clusters and the “King models” and the study by the Toomre & Toomre in 1973 on models for interacting galaxies.

Table 1. ADS Citations up to end 2000 of important papers

Author(s)	Year	Reference	Subject	ADS citations		
				Tot.	9/0	yr <sup>-1</sup>
Schönberg & Chandrasekhar	1942	ApJ <b>96</b> , 161	H-burning and S-C core	57	5	1.0
Baade	1944	ApJ <b>100</b> , 137	Stellar populations	129	5	2.3
de Vaucouleurs	1948	An'Ap <b>11</b> , 247	$R^{1/4}$ -law	388	42	7.5
Spitzer & Schwarzschild	1951	ApJ <b>114</b> , 385	Secular evolution of stellar motions	111	5	2.3
Sandage & Schwarzschild	1952	ApJ <b>116</b> , 463	Giant-branch evolution; MS turn-off	55	6	1.1
Hoyle & Schwarzschild	1955	ApJSuppl <b>2</b> , 1	Giant-branch evolution; HR-diagrams	87	5	1.9
Salpeter	1955	ApJ <b>121</b> , 161	Salpeter-function	922	154	20.5
Burbidges, Fowler & Hoyle	1957	RMP <b>29</b> , 547	Stellar nucleosynthesis	343	15	8.0
Oort, Kerr & Westerhout	1958	MN <b>118</b> , 379	HI structure of the Galaxy	40	2	2.0
Schmidt	1959	ApJ <b>129</b> , 243	SF; "Schmidt-law"	376	39	9.2
King	1962	AJ <b>67</b> , 471	King law	440	42	11.6
Sandage	1962	ApJ <b>135</b> , 333	NGC188 and chemical evolution	98	–	2.6
Eggen, Lynden-Bell & Sandage	1962	ApJ <b>136</b> , 748	Collapse of the Galaxy	670	61	17.6
Schmidt	1963	ApJ <b>137</b> , 758	G-dwarf problem	227	10	6.1
Toomre	1964	ApJ <b>139</b> , 1217	Local disk stability	607	83	16.9
Lin & Shu	1964	ApJ <b>140</b> , 646	Density wave theory	232	18	6.4
Goldreich & Lynden-Bell	1965	MN <b>130</b> , 97	Disk instability and spiral structure	128	14	3.7
King	1965	MN <b>130</b> , 125		195	27	5.6
King	1966	AJ <b>71</b> , 64	King models	803	46	23.6
Lynden-Bell	1967	MN <b>136</b> , 101	Violent relaxation	347	25	10.5
Schmidt	1968	ApJ <b>151</b> , 393	Quasars; $V/V_m$ -test	504	34	15.8
Tinsley	1968	ApJ <b>151</b> , 547	Photometric evolution	66	5	2.1
Freeman	1970	ApJ <b>160</b> , 811	Exponential disks	820	84	27.3
Freeman, Sandage & Stokes	1970	ApJ <b>160</b> , 831	Origin of Hubble types	232	11	7.7
Searle	1971	ApJ <b>168</b> , 327	Disk abundance gradients	349	10	12.0
Peebles	1971	A&A <b>11</b> , 377	Origin of angular momentum	54	5	1.9
Toomre & Toomre	1972	ApJ <b>178</b> , 623	Interacting galaxies	785	90	28.0
Searle, Sargent & Bagnuolo	1973	ApJ <b>179</b> , 427	Photometric evolution of galaxies	337	20	12.5
Ostriker & Peebles	1973	ApJ <b>186</b> , 467	Disk stability and dark halos	439	29	16.3
Tully & Fisher	1977	A&A <b>54</b> , 66	TF-relation	547	63	23.8
Wielen	1977	A&A <b>60</b> , 263	Secular evolution of stellar motions	220	29	9.6
Larson & Tinsley	1978	ApJ <b>219</b> , 46	Photometric evolution	554	29	25.2
Searle & Zinn	1978	ApJ <b>225</b> , 357	Globular clusters and halo formation	539	58	24.5
Tinsley	1980	FCP <b>5</b> , 287	Photometric and chemical evolution	449	57	15.0
van der Kruit & Searle	1981	A&A <b>95</b> , 105	3-D galaxy disk model	280	20	14.8
Gilmore & Reid	1983	MN <b>202</b> , 1025	Galactic thick disk	291	16	17.1

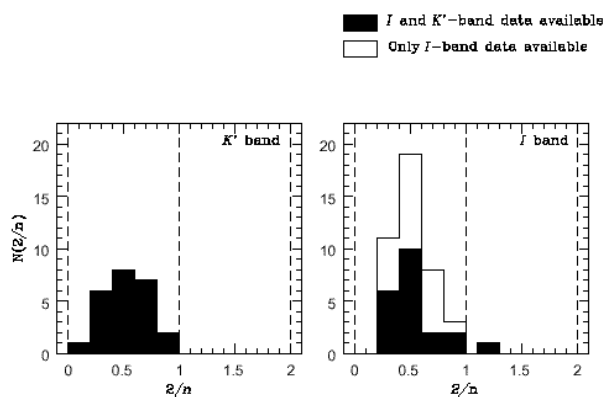


Figure 1. The distribution in  $K'$  and  $I$  of the exponent  $2/n$  in eq. (2) for a sample of edge-on galaxies (from de Grijs *et al.*, 1997).

To complete the search I also checked in ADS which papers between 1945 and 1975 were annually the most highly cited papers in Ap.J. (main journal), A.J., M.N.R.A.S. and A&A. One that comes close is the galaxy redshift survey of Humason *et al.* (1956) (754 citations, 16 in 1999+2000). Outside the field of galaxies there are two papers that clearly exceed it, namely Shakura & Sunyaev (1973; A&A **24**, 337; 1823 citations and 313 in 1999+2000) on the appearance of black holes in binary systems and Nino Panagia (1973, AJ **78**, 929; 1026 and 46) on parameters of early type stars, while Brocklehurst (1971, MN **153**, 471; 787 and 28) on hydrogen population levels in gaseous nebulae comes close.

In any case, Ken's 1970 paper is well within the absolute top ten of citation scores of papers in astronomy and deservedly so.

## 2. Properties of disks

The exponential nature of the light distribution in galactic disks was extended to a three-dimensional model using the self-gravitating isothermal sheet description for the vertical distribution (van der Kruit & Searle, 1981)<sup>1</sup>:

$$L(R, z) = L(0, 0) e^{-R/h_R} \operatorname{sech}^2 \left( \frac{z}{z_0} \right). \quad (1)$$

The isothermal assumption was later dropped and replaced by the family of models (van der Kruit, 1988)

$$L(R, z) = L(0, 0) e^{-R/h_R} \operatorname{sech}^{2/n} \left( \frac{nz}{2h_z} \right). \quad (2)$$

<sup>1</sup>As explained in that paper, the idea to use the isothermal sheet was inspired by a remark made by Ken Freeman during IAU Symposium **77** in 1978.

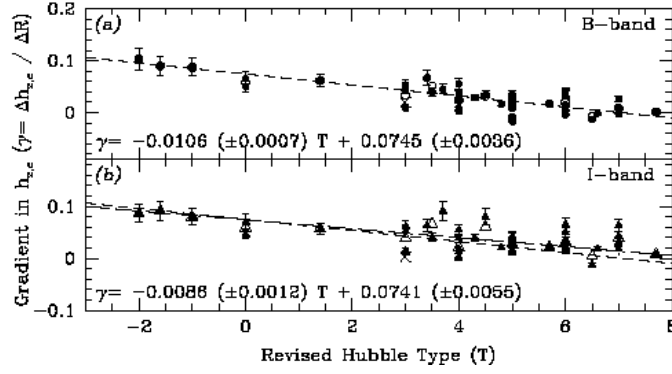


Figure 2. The radial change in the vertical scaleheight in a sample of edge-on galaxies (change in kpc in  $h_z$  per kpc in  $R$ ) as a function of morphological type. Except for the earliest types (S0 to Sab) the change is very small (from de Grijs & Peletier, 1997).

This ranges from the isothermal distribution ( $n = 1$ ) to the exponential function ( $n = \infty$ ). From actual fits in  $I$  and  $K'$  de Grijs *et al.* (1997) found (Fig. 1)

$$\frac{2}{n} = 0.54 \pm 0.20. \quad (3)$$

I will take the whole range from the sech-function to the exponential (that is  $n = 2 - \infty$ ;  $2/n = 0 - 1$ ) into account in what follows, which is reflected in the “uncertainties” in the coefficients in the equations below.

The constancy of the vertical scaleheight  $h_z$  with radius has been studied in detail by de Grijs & Peletier (1997) (see Fig. 2) in the optical and near-IR and been confirmed at least for late-type galaxies. Disks in early type disks might have a small variation of  $h_z$  with radius.

With a constant mass-to-light ratio  $M/L$ , the luminosity density  $L(R, z)$  is proportional to the space density  $\rho(R, z)$ . Then the surface density becomes

$$\Sigma(R) = (2.6 \pm 0.6)\rho(R, 0)h_z, \quad (4)$$

the vertical velocity dispersion in the plane

$$(\sigma_z)_{z=0} = \sqrt{(4.0 \pm 0.9)G\Sigma(R)h_z}, \quad (5)$$

and the z-velocity dispersion integrated perpendicular to the plane

$$\sigma_z = \sqrt{(5.0 \pm 0.2)G\Sigma(R)h_z}. \quad (6)$$

For the vertical velocity dispersion of the stars we expect for a constant  $h_z$

$$\sigma_z^2 \propto e^{-R/2h_R}. \quad (7)$$

This is consistent with observations by van der Kruit & Freeman (1986) and in Roelof Bottema’s thesis (1995, see also 1993), at least out to about 2 scalelengths

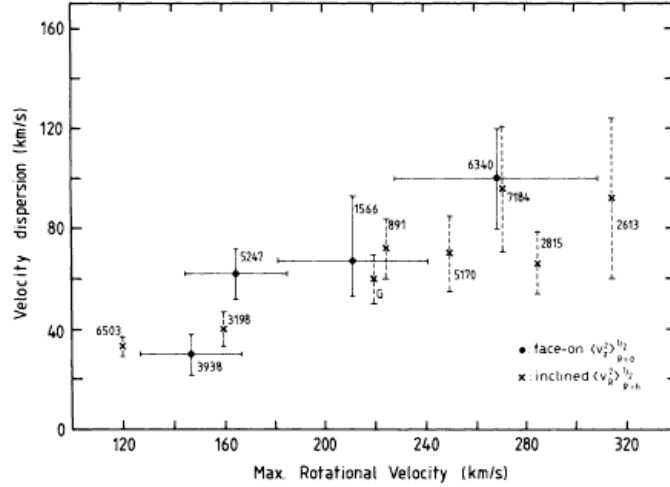


Figure 3. The relation between the stellar velocity dispersion and the maximum rotation velocity in disks of galaxies. The velocity dispersion is the vertical one at the center for face-on systems and the radial one at one scalelength in highly inclined systems (from Bottema, 1993).

(in  $B$ ). Furthermore, Rob Swaters in his thesis (1999; chapter 7) found it also in the late-type dwarf UGC 4325, again out to about 2 scalelengths. On the other hand, Gerssen *et al.* (1997) could not confirm it in NGC 488: they suggest that the scalelength in  $B$  is not representative for the mass distribution. Also, NGC 488 is of early type and may not have a constant  $h_z$ . So, the verification of eq. (7) is not complete. It is important to obtain  $K$ -band surface photometry for NGC 488 and the other galaxies in Bottema's sample.

Bottema also found from a sample of 12 galaxies that the (extrapolated) vertical dispersion at the center and the radial one at one scalelength

$$\sigma_{z,0} \sim \sigma_{R,h_R} = (0.29 \pm 0.10)V_{\max}, \quad (8)$$

where  $V_{\max}$  is the rotation velocity in the flat part of the rotation curve (Fig. 3). Even the late-type dwarf UGC 4325 was found to follow this relation (Swaters, 1999;  $\sigma_{R,h_R} \sim 20$  km/s,  $V_{\max} \sim 90$  km/s). It probably arises as follows:

Take the equation for the Toomre (1964) parameter  $Q$  for local stability

$$Q = \frac{\sigma_R \kappa}{3.36 G \Sigma}. \quad (9)$$

Use a Tully-Fisher relation  $L \propto V_{\max}^n$  with  $n = 4$  and note that for a flat rotation curve the epicyclic frequency  $\kappa = \sqrt{2}V_{\max}/R$ . Then (9) can be rewritten as

$$\sigma_{R,h_R} \propto Q \left( \frac{M}{L} \right) \sqrt{\mu_o} V_{\max}. \quad (10)$$

Here  $\mu_o$  is the central face-on surface brightness in  $L_{\odot} \text{pc}^{-2}$ ;  $\mu_o$  and  $M/L$  refer to the old disk population. So the Bottema relation implies that

$$Q \left( \frac{M}{L} \right) \sqrt{\mu_o} \sim \text{constant}, \quad (11)$$

even for low surface brightness dwarfs. Observations of course allow a substantial scatter in this quotient (or the individual terms) among galaxies.

For “normal” disks obeying “Freeman’s law”  $\mu_o$  translates into 21.7 B-mag arcsec<sup>-2</sup> and this then implies

$$Q \left( \frac{M}{L} \right)_B \sim 6. \quad (12)$$

If we ignore for the moment the (dynamical) influence of the gas, it can be shown (van der Kruit & de Grijs, 1999) that at  $R = 1h_R$

$$\sigma_{R,h_R} = (0.48 \pm 0.02) Q \frac{\sigma_{z,h_R}^2}{V_{\max}} \frac{h_R}{h_z}. \quad (13)$$

Here again the “uncertainty” in the coefficient relates to the range of vertical density distributions above. With the Bottema relation (8) this reduces to

$$\left( \frac{\sigma_z}{\sigma_R} \right)_{R=h_R}^2 = \frac{(7.2 \pm 2.5)}{Q} \frac{h_z}{h_R}. \quad (14)$$

In the solar neighborhood the axis ratio of the velocity ellipsoid  $\sigma_z/\sigma_R \sim 0.5$  (Dehnen & Binney, 1998). If this also holds at  $R = 1h_R$  and using  $h_z \sim 0.35$  kpc and  $h_R \sim 4$  kpc, it follows that

$$Q \sim 2.5. \quad (15)$$

The HI-layer thickness can be used to estimate the disk surface density. HI observations of face-on galaxies (e.g. van der Kruit & Shostak, 1984) indicate an HI velocity dispersion  $\sigma_{\text{HI}} = 8 - 10$  km s<sup>-1</sup>. We can write (van der Kruit, 1981)

$$(\text{FWHM})_{\text{HI}} = (2.8 \pm 0.2) \sigma_{\text{HI}} \sqrt{\frac{h_z}{2\pi G \Sigma(R)}}. \quad (16)$$

Another estimate of the disk mass  $M_D$  follows from the global stability criterion of Efstathiou *et al.* (1982)

$$Y = V_{\max} \sqrt{\frac{h_R}{GM_D}} \sim 1.1. \quad (17)$$

For a pure exponential disk the maximum in the rotation curve (Freeman, 1970) occurs at  $R \sim 2.2h_R$  with an amplitude

$$V_{\max}^{\text{disk}} = 0.88 \sqrt{\pi G \Sigma(0) h_R}. \quad (18)$$

Hydrostatic equilibrium at the center gives

$$\sigma_{z,0}^2 = (5.0 \pm 0.2) G \Sigma(0) h_z. \quad (19)$$

Eliminating  $\Sigma(0)$  between (18) and (19) and using the Bottema relation (8) gives

$$\frac{V_{\max}^{\text{disk}}}{V_{\max}} = (0.21 \pm 0.08) \sqrt{\frac{h_R}{h_z}}. \quad (20)$$

So, the flattening of the disk can be used to test the “maximum disk hypothesis”, for which fits to rotation curves usually give about 0.8 to 0.9 for this ratio.

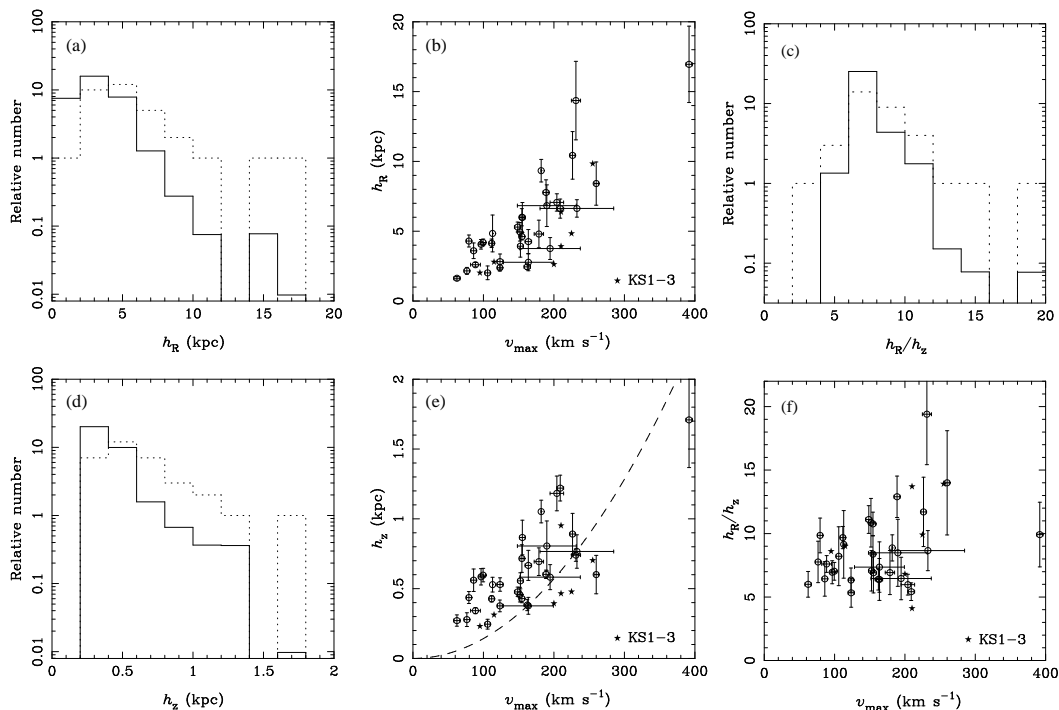


Figure 4. The radial scalelength  $h_R$  and vertical scaleheight  $h_z$  for a sample of 34 edge-on galaxies. Panels (a) and (b) show the distribution (solid line is corrected for sample selection) of  $h_R$  and the correlation with the rotation velocity. Panels (d) and (e) show the same for  $h_z$  and (c) and (f) for the ratio  $h_R/h_z$  (after Kregel *et al.*, 2001).

### 3. Observations of disks

The most extensive study of the photometric disk parameters in the optical and near-IR is that of a statistically complete sample of 86 disk dominated galaxies in Roelof de Jong's thesis (1995, 1996abc; de Jong & van der Kruit, 1994). Some of his conclusions are:

- Freeman's law is really an upper limit to the central surface brightness.
- The scalelength  $h_R$  does not correlate with Hubble type.
- In disks fainter regions are generally bluer, probably resulting from a combination of stellar age and metallicity gradients.
- Outer regions are on average younger and of lower abundance.

Richard de Grijs (1997, 1998; de Grijs & van der Kruit, 1996) presented optical and near-IR surface photometry of a sample of 47 edge-on galaxies. The data have recently been re-analysed by Kregel *et al.* (2001; see also de Grijs *et al.*, 2001) with a new and improved 2-D fitting procedure. Some results (Fig. 4 and 5):

- Both  $h_R$  and  $h_z$  correlate in general terms with  $V_{\max}$ .
- For  $h_R$  this is expected from the Tully-Fisher relation.
- Our Galaxy would be somewhat unusual if the scalelength  $h_R$  is as small as 2

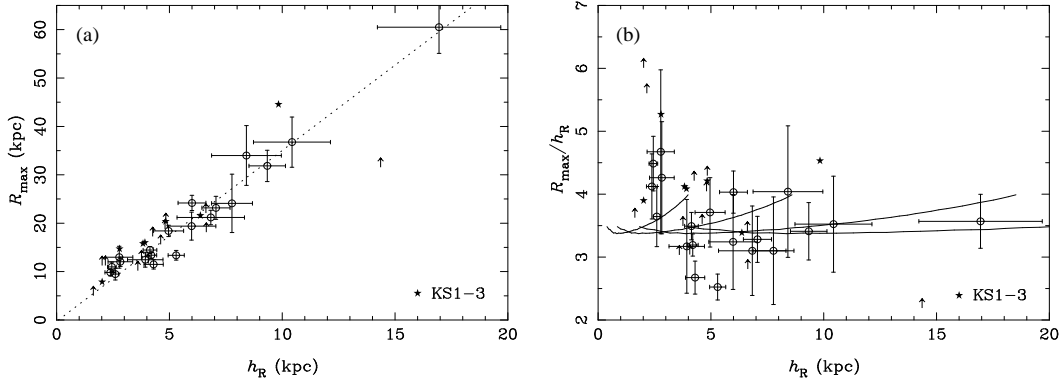


Figure 5. The relation between the truncation radius  $R_{\max}$  and the radial scalelength  $h_{\text{R}}$  of the disks in a sample of edge-on galaxies. The righthand panel shows the ratio  $R_{\max}/h_{\text{R}}$  as a function of  $h_{\text{R}}$  and the lines are models based on the formalism of Dalcanton *et al.*, 1997 (from Kregel *et al.*, 2001).

to 2.5 kpc as some recent studies claim (see also van der Kruit, 2000).

- The flattening of the sample after volume correction is  $h_{\text{R}}/h_{\text{z}} = 7.3 \pm 2.2$ . Then

$$\frac{V_{\max}^{\text{disk}}}{V_{\max}} = 0.57 \pm 0.22. \quad (21)$$

So most galaxies appear not to be “maximum disk”. Recall that this result follows directly from the observations using only the rotation curve of the self-gravitating exponential disk, hydrostatic equilibrium and Bottema’s (empirical, but explainable) relation (8). Bottema (1993) derived a similar result in the analysis of his sample of galaxies, in which he measured the stellar kinematics and found  $V_{\max}^{\text{disk}}/V_{\max} = 0.63 \pm 0.17$ .

- At least 20 of the spirals show radial truncations.
- The ratio of the truncation radius and the scalelength is  $R_{\max}/h_{\text{R}} = 3.6 \pm 0.6$ .
- But large galaxies have smaller values for this ratio than small ones.
- For common disks with scalelengths of 5 kpc or less, the ratio is about 4.

The truncation radius in a simple view results from the maximum specific angular momentum of the sphere from which the disk collapsed. Van der Kruit (1987), in the context the Fall & Efstathiou (1980) picture of disk galaxy formation, then predicted a value of 4.5 for the ratio, based on a Peebles (1971) spin parameter  $\lambda = |E|^{1/2} G^{-1} M^{-5/2}$  of 0.7. Dalcanton *et al.* (1997) have extended this to a models with a dispersion in the spin parameter. We have calculated model surface density profiles with their method for  $M_{\text{tot}} = 10^{10} - 10^{13} M_{\odot}$  and  $\lambda = 0.01 - 0.28$ . These are the lines in Fig. 5b.

For completeness I mention that in many cases there is a warp in the HI-layer in the outer parts, often starting roughly at the truncation radius. This suggests that the warp material has been accreted subsequent to disk formation.



#### 4. IC 5249

The “superthin” disk galaxy IC 5249 has recently been studied by van der Kruit *et al.* (2001). In this paper, we re-analyse the ATCA HI observations reported in Abe *et al.* (1999), use the photometry of three Ken Freeman students, Claude Carignan (1983), Richard Wainscoat (1986) and Yong-Ik Byun (1992, 1998), and present a measurement of the stellar kinematics.

The HI observations show –contrary to the conclusions of Abe *et al.*– a rotation curve that is flat over a large part of the disk at  $V_{\max} = 105 \pm 5 \text{ km s}^{-1}$ . From the available photometry we adopt  $h_R = 7 \pm 1 \text{ kpc}$ ,  $h_z = 0.65 \pm 0.05 \text{ kpc}$ ,  $\mu_o \sim 24.5 \text{ B-mag arcsec}^{-2}$  and  $R_{\max} = 17 \pm 1 \text{ kpc}$ . Note that  $h_z$  is larger than that in our Galaxy! It follows that IC 5249 appears on the sky as a very thin disk, because it combines a low surface brightness with a very long scalelength and a truncation radius after only about 2.5 scalelengths.

From our data we can derive various kinematical and dynamical properties, using the equations given above.

Table 2. Properties of IC 5249 (top part observed, lower part derived)

	$R = 7 \text{ kpc}$	$R = 17 \text{ kpc}$
$\sigma_\theta$ (km/s)	25-30	–
$V_{\text{rot}}$ (km/s)	$90 \pm 5$	$105 \pm 5$
$dV_{\text{rot}}/dR$ (km/s.kpc)	$3 \pm 4$	$0 \pm 1$
$\kappa$ (km/s.kpc)	$20 \pm 6$	$3.5 \pm 0.4$
$\sigma_R$ (km/s)	$35 \pm 5$	–
Asym. drift (km/s)	$10 \pm 3$	–
$\sigma_R$ (km/s)	–	$25 \pm 5$
$\Sigma$ ( $M_\odot \text{ pc}^{-2}$ )	$\sim 25$	$\sim 6$
$\sigma_z$ (km/s)	$19 \pm 4$	$11 \pm 2$
$Q$	$\sim 2$	$\sim 2$
(FWHM) <sub>HI</sub> (kpc)	$0.60 \pm 0.17$	$1.5 \pm 0.5$

At  $R = 7 \text{ kpc}$  the stellar velocity dispersions are similar to the solar neighborhood, while the surface density of the disk is about half that of the solar neighborhood. Star formation must have proceeded much slower in IC 5249 in order to give the low surface brightness. But, surprisingly, as much dynamical evolution has occurred as in the Galactic disk. Also note that with  $\sigma_{R,h}/V_{\max} = 0.33 \pm 0.05$ , IC 5249 falls on the Bottema relation (8).

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## Discussion

*King:* One of the tightest correlations you showed was between  $R_{\max}$  and  $h_R$ . What does  $R_{\max}$  mean, and is there any obvious reason for such a tight correlation?

*van der Kruit:*  $R_{\max}$  is the truncation radius. The tightness of the correlation results from the fact that over scalelengths in the range from 1 to 15 kpc or so, the truncation radius occurs at 3 – 4 scalelengths. I have commented on this in the paper above.

*Byun:* In the 1970 Freeman paper, disk galaxies were divided into two groups, type I and type II. Type II galaxies had exponential disks which *do not* continue into the centers. What are our current understandings of these type II galaxies?

*van der Kruit:* Although there still are some systems having profiles that were called type II, it appears that these no longer make up a substantial fraction of the observed profiles. The prime example, M83, is barred and the bar may be the cause of this behavior, but there are also galaxies that are unbarred and display the type II characteristic. But again it no longer is an important class in modern surface photometry profiles.

*Fall:* My impression is that realistic angular momentum distributions  $M(h)$  produced by hierarchical clustering do not have the sharp features that would lead to edges in the disks. My preferred explanation for the edges at  $R_{\text{edge}} \approx 4r_{\text{disk}}$  is in terms of local instabilities in the disks (Goldreich & Lynden-Bell and Toomre-type instabilities<sup>2</sup>). George Efstathiou and I showed these would give rise to  $R_{\text{edge}} \approx 4r_{\text{disk}}$ .

*van der Kruit:* I am aware of the Fall & Efstathiou (1980) explanation. Actually, in my third paper with Leonard Searle on edge-on galaxies (A&A, 110, 61, 1981) we refer to this and find that it predicts truncations at  $0.8 \pm 0.2$  times the observed ones for our sample.

*van der Kruit:* After the session Mike Fall suggested that the conclusion that the maximum disk hypothesis does not apply to most galaxies using the disk flattening may be affected by underestimating the scaleheight due to the presence of stellar generations in the disks with a range in vertical velocity dispersions and scaleheights. The younger of these are brighter and their smaller dispersions lead to a lower scaleheight in the photometry than in the mass distribution. It is well-known that the observed diffusion of stellar random motions gives –according to observations in the solar neighborhood– a velocity dispersion proportional to  $\sqrt{\text{age}}$  and Mike points out that the stellar generations therefore have a scaleheight proportional to age. I was urged by some participants to reply to this. According to eq. (20) a systematic underestimate of the scaleheight  $h_z$  results in an *overestimate* in the ratio  $V_{\max}^{\text{disk}}/V_{\max}$  and allowing for it then takes the disks even further from maximum disk. However, the effect must also occur for the observed velocity dispersions in face-on disks, so Bottema would systematically have underestimated the dispersions resulting in an underestimate of the coefficient in eq. (8) and (20). I have returned to some old notes to myself on this and updated these. In view of the potential importance of the effect I document

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<sup>2</sup>The references to the papers by Goldreich & Lynden-Bell (1965) and Toomre (1964) appear in Table 1 [PCvdK]

this here.

The luminosity of a generation of stars as a function of age can be estimated in three ways. First look at the luminosity of the main sequence (MS) turn-off stars. The stellar MS luminosity is roughly proportional to  $M^3$  and MS lifetime  $\tau$  to  $M^{-2}$ . So luminosities  $L_{\text{MS}}$  are roughly proportional to  $\tau^{-3/2}$ . Secondly, we may look at the giants only. I estimate from evolutionary tracks that at the tip of the giant branch the luminosity  $L_{\text{Giant}}$  is crudely proportional to the mass on the MS (at least for stars between one and a few solar masses; MS lifetimes of 1 to 10 Gyr), so that  $L_{\text{Giant}} \propto \tau^{-1/2}$ . Finally, single burst models of photometric evolution can be used. A nice example is illustrated in Binney & Merrifield (Galactic Structure; Princeton Univ. Press, 1998) in Fig. 5.19 on page 318. The  $(M/L)_B$  is proportional to age, so that the luminosity of a single burst  $L_{\text{SB}} \propto \tau^{-1}$ . This is –as expected– nicely in between  $L_{\text{MS}}$  and  $L_{\text{Giant}}$ .

To estimate the effect it is most practical to look at the integrated velocity dispersion in a face-on disk. The weighted velocity dispersion can then be estimated by integrating over the relevant ages  $\tau$  in

$$\langle \sigma_z^2 \rangle = \frac{\int \text{SFR}(\tau) L(\tau) \sigma^2(\tau) d\tau}{\int \text{SFR}(\tau) L(\tau) d\tau}.$$

To estimate the error this should then be compared to the case of equal weighing of the generations

$$\langle \sigma_z^2 \rangle = \frac{\int \text{SFR}(\tau) \sigma^2(\tau) d\tau}{\int \text{SFR}(\tau) d\tau}.$$

We had  $\sigma^2 \propto \tau$  and may take for late type disks as a reasonable approximation a constant star formation rate  $\text{SFR}(\tau)$ .

In order to check the effect on the scaleheights the best approximation is to use  $L_{\text{SB}}$ . The results depend on the range in  $\tau$  we take to perform the integration. Since fits for  $h_z$  are made above the dust lanes we need to ignore the youngest generations. As examples I take the integration from 2 and 3 to 10 Gyr ; this means ignoring generations in which the scaleheights are less than 0.2 and 0.3 times that of the oldest generations. The values for  $\sqrt{\langle \sigma \rangle^2}$  are then underestimated by respectively 9 and 6% and the scaleheights by 17 and 11%.

For the effect on the velocity dispersions we have to take  $L_{\text{Giant}}$ , since these are measured by comparing to a late-GIII or early-KIII template star. The galaxy spectra refer in practice for this mostly to interarm regions, so it is fair to take the lower limit in the integration now at 1 or 2 Gyr. Then  $\sqrt{\langle \sigma \rangle^2}$  is underestimated by respectively 8 and 4%. Even if the lower integration limit is set to zero (assuming red supergiants fit the luminosity class III template star), the error is only 18%.

So, the effects are small and furthermore in an application of eq. (20) we need to take the square root of the scaleheight, while the two effects work in opposite directions.