THE MASSES OF DISKS

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Outline

Background

Why do we want to know the disk mass?

- The mass-to-light ratio M/L contains important information on star formation and the Initial Mass Function.
- The properties (mass and flattening) of the dark halo depend on the contribution of the disk mass to the rotation curve.
- The disk surface density $\Sigma(R) = \Sigma_\circ \exp\left(-R/h\right)$ or mass $M_{\rm disk}$ play a role in stability criteria, in particular:
  - Local stability[1]:

$$Q = \frac{\sigma_{\rm R}\kappa}{3.36\,G\Sigma}$$

  - Global disk stability[2]:

$$Y = V_{\rm rot}\left(\frac{h}{GM_{\rm disk}}\right)^{1/2} \gtrsim 1.1$$

[1] A. Toomre, *Ap.J.* 139, 1217 (1964)
[2] G. Efstathiou, G. Lake & J. Negroponte, *MNRAS* 199, 1069 (1982)

Maximum disk hypothesis

The discovery of flat rotation curves[3] implied dark matter, but it is not a priori obvious to what extent this is in the disk.

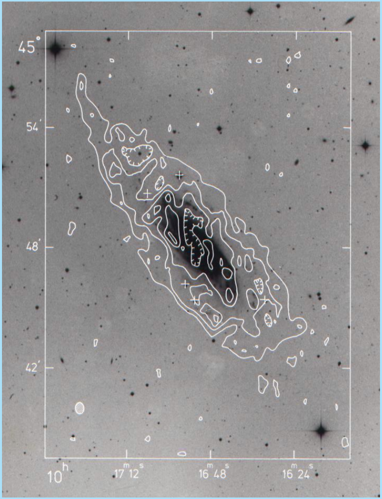
The maximum disk hypothesis was designed to optimize the mass in the disk, which was esentially unconstrained by analysis of the rotation curve only.

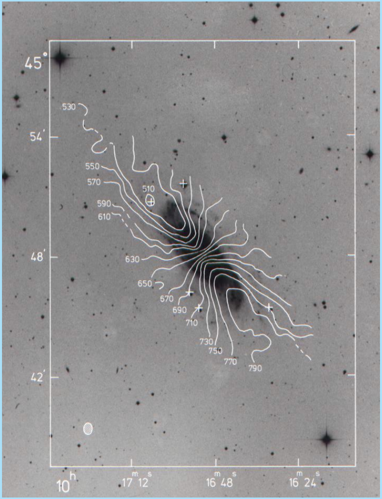
It was based on an analysis of the rotation curve of NGC 3198[4], which has essentially no bulge.

The HI extends out to 11 scalelengths on this long WSRT integration.

[3] V. Rubin & W.K. Ford, Ap.J. 159, 379 (1970); see also M.S. Roberts, ASPO Conf. Ser. 395, 283 (2008).

[4] T.S. van Albada, J.N. Bahcall, K. Begeman & R. Sancisi, *Ap.J.* 295, 305 (1985)


45°
54′
48′
42′
10h
17m12s
16m48s
16m24s


45°
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670
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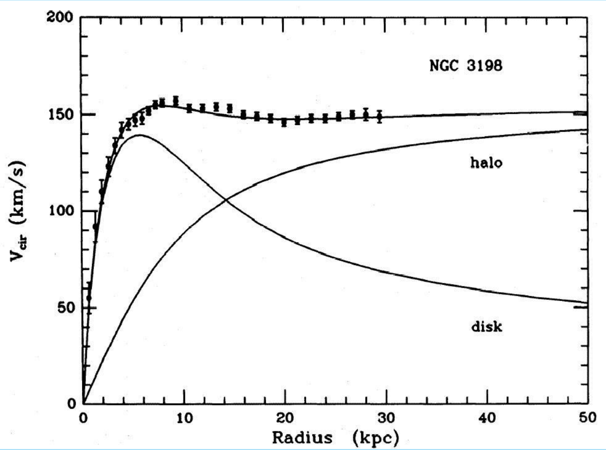
750
770
790
10h
17m12s
16m48s
16m24s

The procedure then is to choose an M/L of the disk that gives the maximum amplitude of the disk rotation curve that is allowed by the observations.

The two free parameters of the dark halo, core radius R_{c} and central density $\rho_{\circ}$ are then used to fit the rotation curve.

This is called the "maximum disk hypothesis", since it is a fit to the rotation curve with the largest amount of mass possible in the disk (and the largest disk M/L).

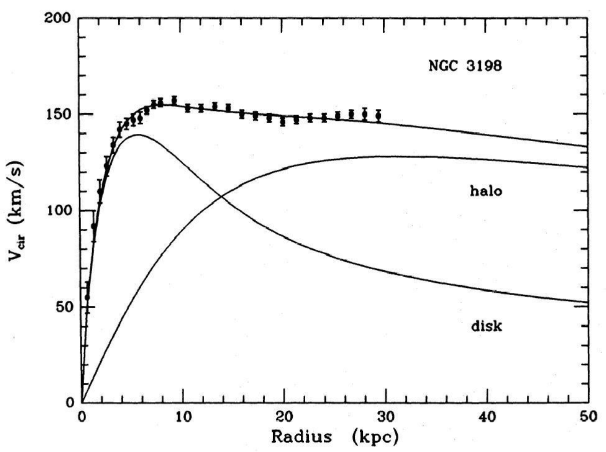
The maximum disk solution to the rotation curve of NGC 3198 looks as follows.



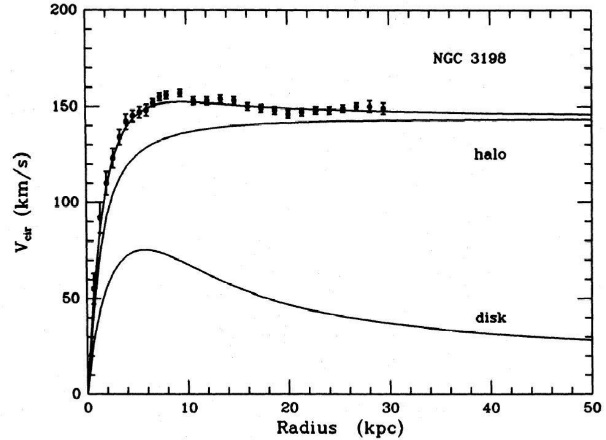
This particular model for NGC 3198 has a total mass of 15×10^{10} $M_\odot$ within 30 kpc.

Within this radius the ratio of dark to visible matter is 3.9. Within the optical edge this ratio is 1.5.

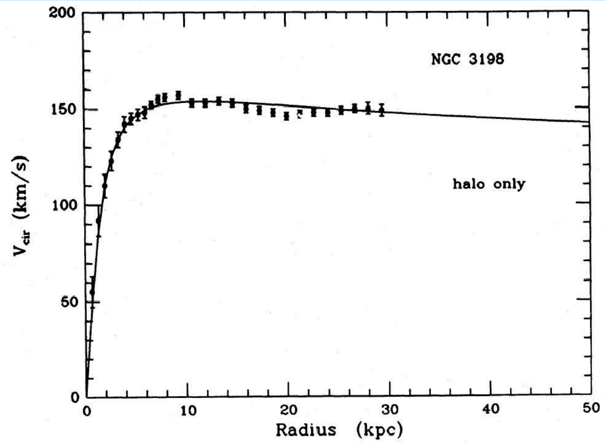
By adjusting the halo parameters one can minimize the dark halo mass by assuming that the rotation curve falls beyond the last measured point.


NGC 3198
halo
disk
V_{cir} (km/s)
Radius (kpc)
0
50
100
150
200
0
10
20
30
40
50

The difficulty with the maximum disk hypothesis is that it is possible to make similar good fits with lower disk masses ...



... and even no disk mass at all!



In maximum disk fits it is found that the ratio of the rotation curve of the disk alone ($V_{\mathrm{rot,disk}}$) to that observed amplitude of the rotation curve[5] is

$$\frac{V_{\mathrm{rot,disk}}}{V_{\mathrm{rot,obs}}} = 0.85 \pm 0.10.$$

The rotation curve of an exponential disk[6] with central surface density $\Sigma_\circ$ and scalelength h has a maximum at $R = 2.2h$ of amplitude

$$V_{\mathrm{rot,disk}} = 0.88(\pi G h \Sigma_\circ)^{1/2}.$$

[5] P.D. Sackett, Ap.J. 483, 103 (1997)
[6] K.C. Freeman, Ap.J. 160, 811 (1970)

Independent checks on the maximum disk hypothesis

There are independent ways in which the maximum disk hypothesis can be checked by independent measurement of M/L.

a. The truncation feature in the rotation curve

The truncation of the (stellar) disk produces in principle a feature in the rotation curve that can be used to estimate the mass of the disk. It has been done in two cases where the mass of the halo within the truncation radius has been estimated:

- NGC 5907[7]: $(M_{\rm halo})_{\rm R_{opt}} \approx 60\%$ (probably not maximum disk)
- NGC 4013[8]: $(M_{\rm halo})_{\rm R_{opt}} \approx 25\%$ (possibly maximum disk)

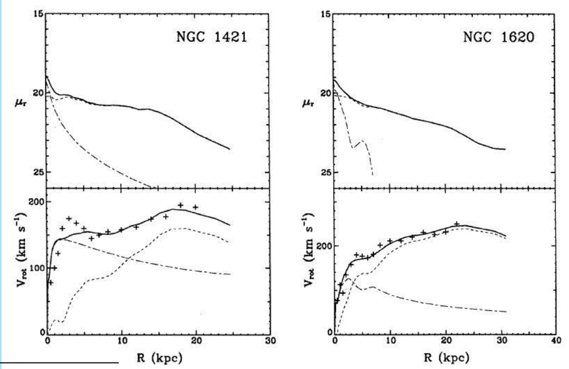
[7] S. Casertano, MNRAS 203, 735 (1983)
[8] R. Bottema, A.&A. 306, 345 (1996)

b. "Wiggles" in rotation curves

The inner parts op rotation curves can often be fit without a dark halo and features in luminosity profiles seem to correspond to features in rotation curves[9].

Top shows the light distributions of disk and bulge.

Bottom shows the rotation curve with constant M/L in both components.

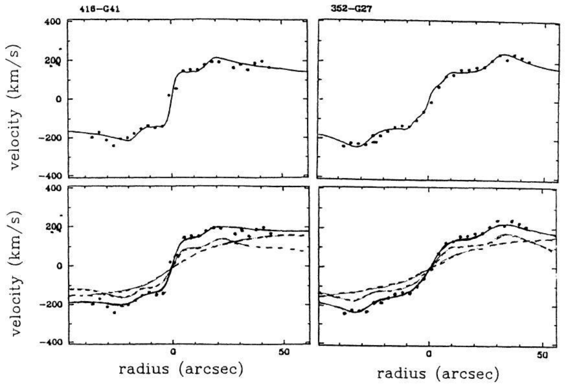


[9] E.g. S. Kent, A.J. 91, 1301 (1986)

This suggests maximum disks, but even if disks are not dynamically dominant in the inner parts the wiggles can still be reproduced.[10]

Top has the rotation curve from the photometry without a dark halo.

Bottom has reduced the disk mass by half and a dark halo added.



[10] P.C. van der Kruit, IAU Symp. 164, 227 (1995)

c. *Maximum rotation versus scalelength*

The following clever argument[11] makes use of the scatter in the Tully-Fisher relation.

For a pure exponential disk the maximum in the rotation curve occurs at $R = 2.2h$ with an amplitude of

$$V_{\max} \propto \sqrt{h\Sigma_\circ} \propto \sqrt{\frac{M_{\rm disk}}{h}}$$

For fixed disk-mass $M_{\rm disk}$ this gives

$$\frac{\partial \log V_{\max}}{\partial \log h} = -0.5$$

[11] S. Courteau & H.-W. Rix, Ap.J. 513, 561 (1999)

So at a given absolute magnitude (or mass) lower scalelength disks should have higher rotation.

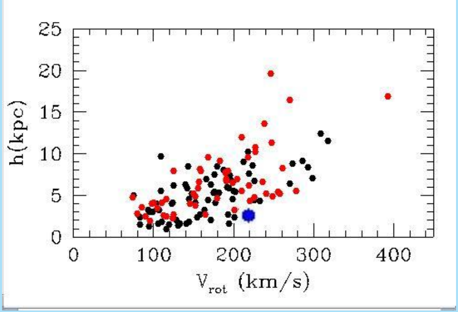
If all galaxies are maximum disk this should be seen in scatter of the Tully-Fisher relation.

This is not observed and the estimate is that on average $V_{\rm disk} \sim 0.6 V_{\rm total}$.

Our Galaxy

My preferred value for the scalelength of the disk[12] is 4 to 5 kpc.

The value often seen in the literature is about 2.5 kpc, but this makes the Galaxy very small for its rotation[13].



[12] P.C. van der Kruit, A&A. 157,230 (1986)

[13] Compilation of data is from van der Kruit, A.S.P. Conf. Ser. 396, 123 (2008)

The measured surface density[14] of the stellar disk in the solar neighbourhood is 50 to 80 $\mathrm{M}_\odot$ pc^{-2}

With this it can be estimated that the luminous matter provides a maximum rotation velocity of 155 ± 30 km/s, while the observed value is 225 ± 10 km/s.
So $\frac{V_{\mathrm{rot,disk}}}{V_{\mathrm{rot,obs}}} = 0.69 \pm 0.14$; the Galaxy is then probably not maximum disk.

However, one can change the parameters within uncertainties to get a result in agreement with maximum disk[15].

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[14] K.H. Kuijken & G. Gilmore, MNRAS 239, 605 (1989) find $46 \pm 9\ \mathrm{M}_\odot \mathrm{pc}^{-2}$ and J.N. Bahcall, Ap.J. 287, 926 (1984) $80 \pm 20 \mathrm{M}_\odot \mathrm{pc}^{-2}$

[15] e.g. J.A. Sellwood & R.H. Sanders, MNRAS 233, 611 (1988); P.D. Sackett, Ap.J. 483, 103 (1997)

Vertical dynamics of stellar disks

Background

The vertical velocity dispersion of the stars in a disk can be combined with the thickness of disk to estimate of the disk surface densities Σ.

The Poisson equation for the case of axial symmetry is

$$\frac{\partial K_{\rm R}}{\partial R} + \frac{K_{\rm R}}{R} + \frac{\partial K_{\rm z}}{\partial z} = -4\pi G \rho(R,z)$$

At small z, the first two terms on the right are equal to $2(A-B)(A+B)$ and this is zero for a flat rotation curve. So we may use the plane-parallel case:

$$\frac{dK_{\rm z}}{dz} = -4\pi G \rho(z)$$

The hydrodynamic equation for the axi-symmetric case is

$$\frac{d}{dz}\left[\rho(z)\sigma_{\mathrm{z}}^{2}\right] = \rho(z)K_{\mathrm{z}}$$

For an isothermal distribution this becomes

$$\frac{d\rho(z)}{dz} = \frac{\rho(z)K_{\mathrm{z}}}{\langle V_{\mathrm{z}}^{2}\rangle}$$

The equations for the isothermal sheet are the solutions of this set of equations.

$$\rho(z) = \rho_{\circ}\,\mathrm{sech}^{2}\left(\frac{z}{z_{\circ}}\right) \quad ; \quad \Sigma_{\circ} = 2z_{\circ}\rho_{\circ}$$

$$K_{\mathrm{z}} = -4\pi G\rho_{\circ}z_{\circ}\tanh\left(\frac{z}{z_{\circ}}\right) \quad ; \quad \langle V_{\mathrm{z}}^{2}\rangle^{1/2} = z_{\circ}\sqrt{2\pi G\rho_{\circ}}$$

Disks are not entirely isothermal, since velocity dispersions of the stellar generations increase with age. Therefore replace the solution by the set[16]

$$\rho(z) = 2^{-2/n} \rho_{\rm e}\ \mathrm{sech}^{2/n} \left(\frac{nz}{2z_{\rm e}} \right)$$

Consider the range $n = \infty$ (exponential) through $n = 2$ (sech-distribution) to $n = 1$ (the isothermal sech^2).

Then we can write (using this range of n)

$$\sigma_{\rm z}(R) = (2.3 \pm 0.1)\sqrt{G\Sigma(R) z_{\rm e}}$$

[16] P.C. van der Kruit, A.&A. 192, 117 (1988)

The study of stellar velocity dispersions in disks of galaxies started in the mid-eighties[17] and nineties[18], leading among others to the discovery of a relation between a fiducial velocity dispersion and the integrated magnitude or maximum rotation velocity of a galaxy.

I summarise the work by Kregel et al.[19] using surface photometry (from Richard de Grijs), optical spectroscopy and HI synthesis obervations of a sample 15 edge-on galaxies.

More recent work in disk stellar dynamics in the Disk-Mass Survey will presented later during this symposium.

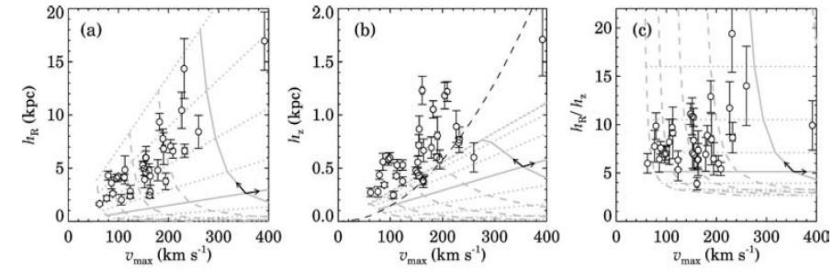
[17] P.C. van der Kruit & K.C. Freeman, Ap.J. 303, 556 (1986)

[18] R. Bottema, A.&A. 275, 16 (1993)

[19] M. Kregel, P.C. van der Kruit & R. de Grijs, MNRAS 334, 646 (2002) and a series of papers ending with M. Kregel, P.C. van der Kruit & K.C. Freeman, MNRAS 358, 503 (2005)

The Kregel et al. study

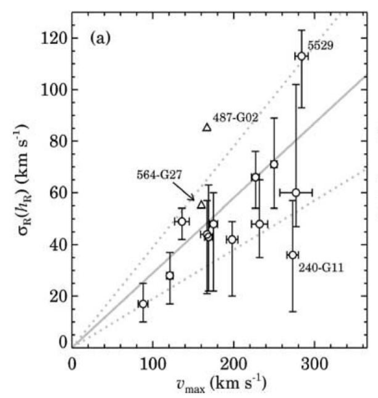
The luminosity scale parameters correlate with the maximum rotation velocity.



$$z_e = (0.45 \pm 0.05) \left(\frac{V_{\max}}{100 \text{ km/s}} \right) - (0.14 \pm 0.07) \text{ kpc}$$

The 'Bottema relation' between radial velocity dispersion at one scalelength versus maximum rotation velocity is confirmed as

$$\sigma_{\mathrm{R}}(h) = (0.29 \pm 0.10) V_{\mathrm{max}}$$



Galaxies with high rotation velocity are larger and thicker and have higher stellar velocity dispersions.

The equation for the rotation curve of the exponential disk gives

$$V_{\rm rot,disk} = (0.69 \pm 0.03)\sigma_{\rm z|R=0}\sqrt{\frac{h}{z_{\rm e}}}$$

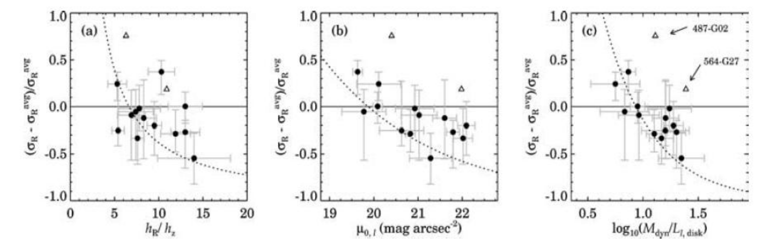
Using the Bottema's empirical relation, $\sigma_{\rm z}/\sigma_{\rm R} = 0.6$, and our observation[20] that the flattening $h/z_{\rm e} = 7.3 \pm 2.2$, we find

$$\frac{V_{\rm rot,disk}}{V_{\rm rot,obs}} = (0.21 \pm 0.08)\sqrt{\frac{h_{\rm R}}{h_{\rm z}}} = 0.57 \pm 0.22$$

Therefore disks are in general not maximal.

[20] M. Kregel, P.C. van der Kruit & R. de Grijs, MNRAS 334, 646 (2002)

The scatter in the (σ-$V_{\rm max}$)-relation correlates with disk flattening, face-on central surface brightness and dynamical mass-to-light ratio[21].



Low surface brightness disk are more flattened and have smaller stellar velocity dispersions.

[21] Dynamical mass is $M_{\rm dyn} = 4hV^2_{\rm max}/G$

The parameter we cannot measure is the axis ratio of the velocity ellipsoid, or in short the 'anisotropy', $\sigma_{\rm z}/\sigma_{\rm R}$.

It can be shown from simple arguments that it is related to the flattening[22] (formally at one scalelength):

$$\frac{\sigma_{\rm z}}{\sigma_{\rm R}} = \left(\frac{7.77}{Q} \frac{z_{\rm e}}{h} \right)^{1/2}$$

It depends on the square root of the flattening and Toomre's Q.

Since $z_{\rm e}/h = 0.1 - 0.3$ and for a $Q \sim 2.5$, as found in simulations[23], we expect an anisotropy of $\sigma_{\rm z}/\sigma_{\rm R} \sim 0.6 - 0.8$.

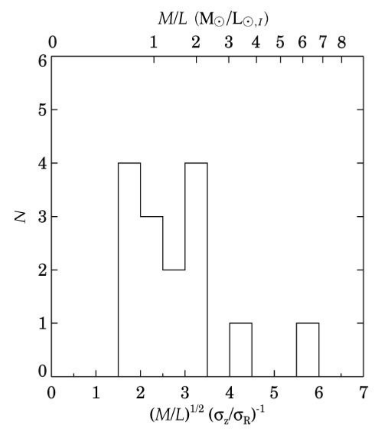
[22] P.C. van der Kruit & R. de Grijs, A.&A. 352, 129 (1999)

[23] J.A. Sellwood & R.G. Carlberg, Ap.J. 282, 61 (1984); E. Athanasoulla & J.A. Sellwood, MNRAS 221, 213 (1986); J.C. Mihos, S.S. McGaugh & W.J.,G. de Block, Ap.J. 477, 79 (1997); R. Bottema, MNRAS 344, 358 (2003)

$$\sqrt{\frac{M}{L}}\left(\frac{\sigma_z}{\sigma_R}\right)^{-1} = 2.7 \pm 0.7$$

At the top-axis M/L for an anisotropy of 0.6.

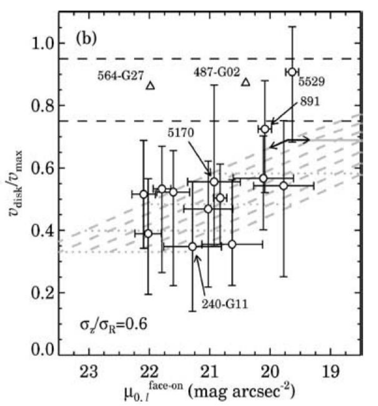
$$\frac{M}{L_I} = 1.2 \pm 0.2$$



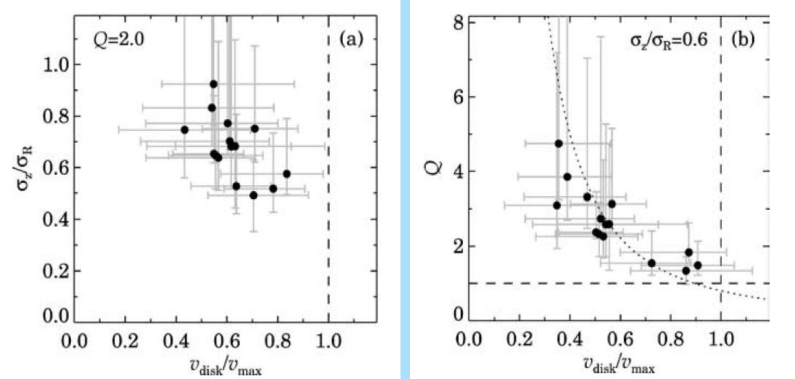
If disks have similar aniotropies the range in M/L is small.

$$\frac{V_{\mathrm{rot,disk}}}{V_{\mathrm{rot,obs}}} = 0.58 \pm 0.18$$

ESO487-G02 and 564-G27 are the high M/L galaxies in the previous figure.



So at least 12 of our disks are probably submaximal. Very high surface density disks may still be maximal.



Maximal disks appear to have more anisotropic velocity dispersions or are less stable according to Q.

Thickness of the HI-layer

Background

The thickness of the gas layer can be used to measure the surface density of the disk independent of the rotation curve.

The density distribution of the exponential, locally isothermal disk was:

$$\rho_*(R,z) = \rho_*(0,0)\, \exp\left(-R/h\right)\, \mathrm{sech}^2(z/z_\circ)$$

If the HI has everywhere[24] a velocity dispersion $\langle V_\mathrm{z}^2\rangle_\mathrm{HI}^{1/2}$, and if the stars dominate the gravitational field

$$\rho_\mathrm{HI}(R,z) = \rho_\mathrm{HI}(R,0)\, \mathrm{sech}^{2p}(z/z_\circ) \quad ; \quad p = \frac{\langle V_\mathrm{z}^2\rangle_*}{\langle V_\mathrm{z}^2\rangle_\mathrm{HI}}$$

[24] e.g. G.S. Shostak & P.C. van der Kruit, A.&A. 132, 20 (1984)

The full width at half maximum is to within 3%

$$W_{\rm HI} = 1.7 \langle V_{\rm z}^2 \rangle_{\rm HI}^{1/2} \left[\frac{\pi G (M/L) \mu_\circ}{z_\circ} \right]^{-1/2} \exp\left(R/2h\right)$$

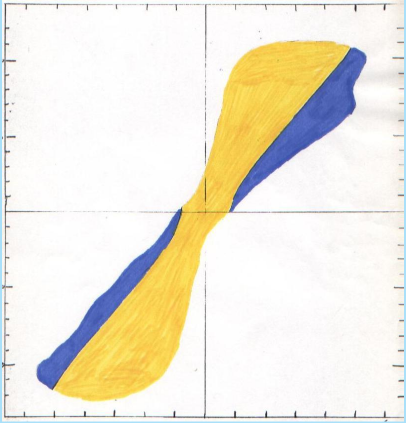
So the gas layer increases exponentially in thickness with an e-folding of $2h$.

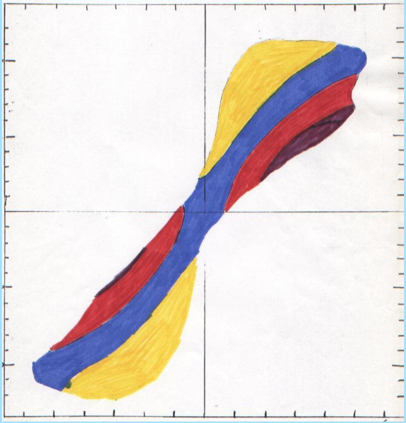
This can be used to model the observed $W_{\rm HI}$ in the position-velocity diagram.

Here are some figures from a study of NGC 891[25].

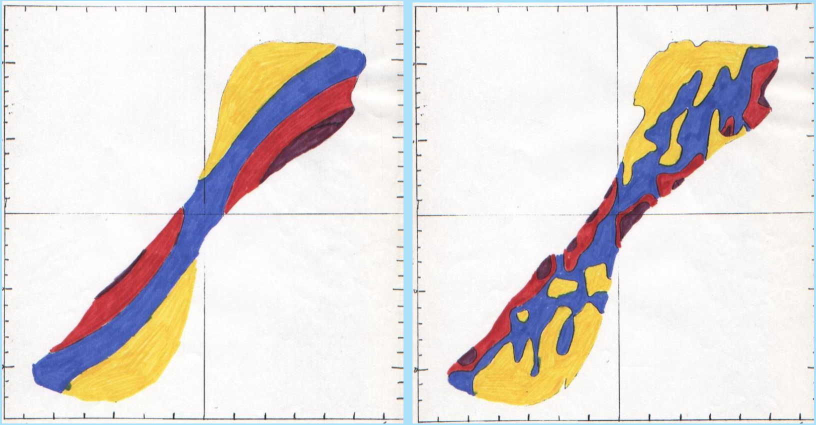
[25] P.C. van der Kruit, A.&A. 99, 298 (1981)

Here is the equivalent width in the (x, V)-diagram for a model with flaring, but for inclinations of 90° (left) and $87\overset{\circ}{.}5$ (right).





This model with an inclination of 87.°5 (right) fitted the observed (x, V)-diagram (left).



This showed that the dark matter is not in the disk.

The modelling indicates[26] that the disk-alone rotation curve of NGC 891 is ~ 140 km/s, while the observed value is 225 ± 10 km/s, so

$$\frac{V_{\mathrm{rot,disk}}}{V_{\mathrm{rot,obs}}} \sim 0.6.$$

In other systems similar values were found; in NGC 4244 the analysis of the flaring[27] indicated a disk-alone rotation of '50 to 100% of that of maximum disk'.

Flaring of the HI-layer in NGC 4244 was used[28] to infer that the dark matter is highly flattened.

[26] P.C. van der Kruit & L. Searle, A.&A. 110, 61 (191982)
[27] R. Olling, A.J. 112, 457 (1996)
[28] R. Olling, A.J. 112, 481 (1995)

The study of O'Brien et al.

The following is from a recent Ph.D. thesis at ANU by Jess O'Brien. It has been submitted as a series of 4 papers[29].

The aim was to meassure the shape of the dark matter halo by determining the force field of the halo vertically using HI-layer flaring and radially from rotation curve decomposition.

This method was optimised by selecting small, HI-rich, late-type edge-on galaxies which often have *low-mass disks* compaerd to their halos (Albert Bosma).

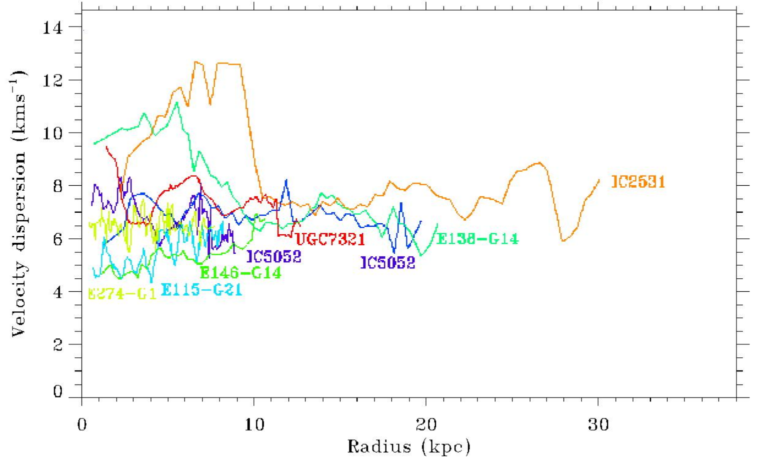
[29] Paper I: J.C. O'Brien, K.C. Freeman, P.C. van der Kruit & A. Bosma, A.&A. submitted (2009); Papers II-IV: J.C. O'Brien, K.C. Freeman & P.C. van der Kruit, A.&A. submitted (2009)

A sample of 8 HI-rich, late-type edge-on galaxies was defined.

Methods to model the (x, V)-diagram have been extended to include at the same time the HI surface density, the rotation curve and the HI velocity dispersion, all as a function of galactocentric radius.

This 'radial decomposition XV modelling method' was extensively tested and applied to HI-data of the sample galaxies (from ATCA or VLA archive).

Here is the velocity dispersion as a function of radius.



Galaxies with $V_{\rm rot} > 120$ km/s have well-defined dustlanes[30].

Since dust scaleheight $h_{\rm dust}$ is related to the ISM velocity dispersion σ as

$$\frac{\sigma^2}{h_{\rm dust}^2} = -\frac{\partial K_{\rm z}}{\partial z} = 4\pi G \frac{\Sigma}{2h_\star}$$

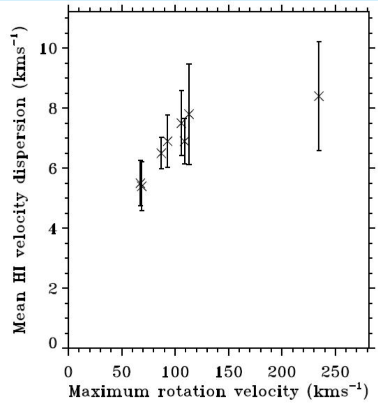
and since Σ and $h_\star$ are both roughly proportional to $V_{\rm rot}$[31], we expect $h_{\rm dust}$ to be proportional to σ.

[30] J.J. Dalcanton, P. Yoachim & R.A. Bernstein, Ap.J. 608, 189 (2004)
[31] S. Gurovich, K.C. Freeman, H. Jerjen & I. Puerari, Ap.J. submitted (2009); M. Kregel, P.C. van der Kruit & K.C. Freeman, MNRAS, 351, 1247 (2004)

The largest galaxy in the figure is IC 2531, which does have a well-defined dustlane.

However, we see no decrease in σ as $V_{\rm rot}$ increases, as would be expected.

So no support for the 'variable turbulence' explanation of Dalcanton et al.

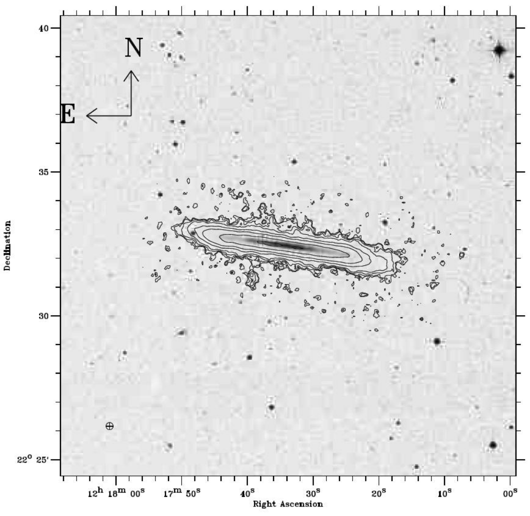


There is a detailed application to UGC 7321 in paper IV of O'Brien et al.

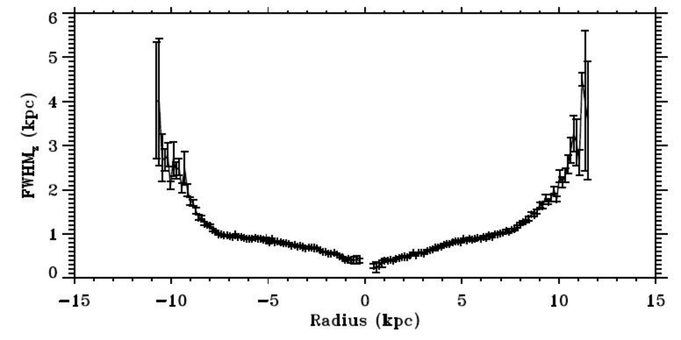
HI-data are from the VLA archive (observations of Lyn Matthews).

Optical data from SDSS (provided by Michael Pohlen)

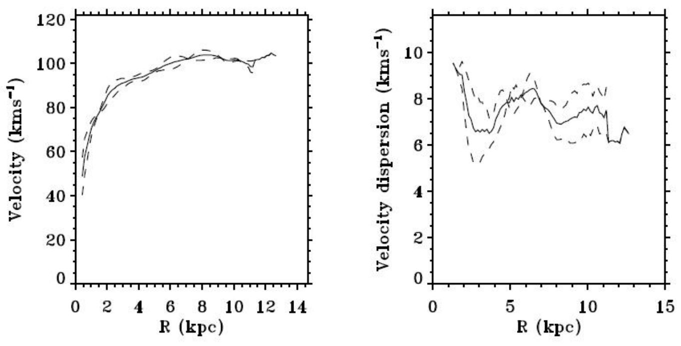
Total HI-map


N
E
40
35
30
22° 25'
Declination
12h 18m 00s
17m 50s
40s
30s
20s
10s
00s
Right Ascension

Here is the deduced flaring of the HI-layer. Note that it is highly symmetric.

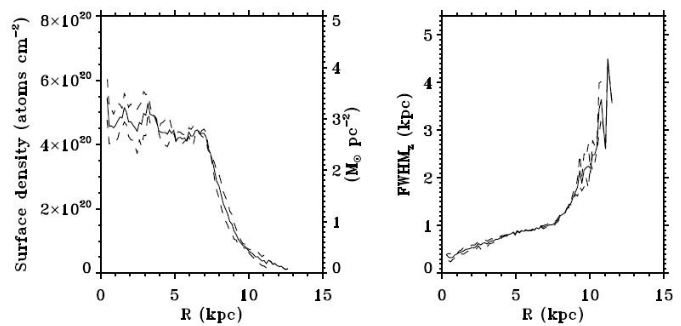


Here we see the rotation curve and velocity disperion profile[32].



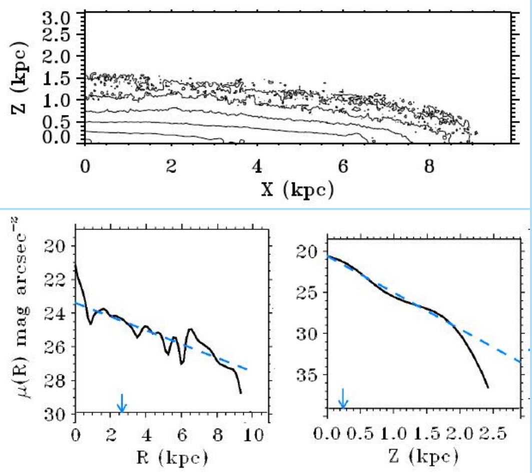
[32] Dashed lines are the two sides separately

And the radial HI surface density profile and the average flaring profile.



And finally the observed surface brightness distribution (top)

and the inferred deprojected radial and vertical luminosity profiles (bottom).



The dark halo forces are modelled using a flattened pseudo-isothermal halo with density distribution[33]

$$\rho(R,z) = \frac{\rho_{\mathrm{h},\circ} R_{\mathrm{c}}^2}{R_{\mathrm{c}}^2 + R^2 + z^2/q^2}$$

The resulting shape of the rotation curve (at $z = 0$) is independent of the flattening $q = c/a$.

The best fit decomposition of the rotation curve then has a disk M/L of 1.05.

This is far from maximum disk and the disk forcefield is small w.r.t. to that of the halo.

[33] P.D. Sackett, H.-W. Rix, B.J. Jarvis & K.C. Freeman, Ap.J. 436, 629 (1994)

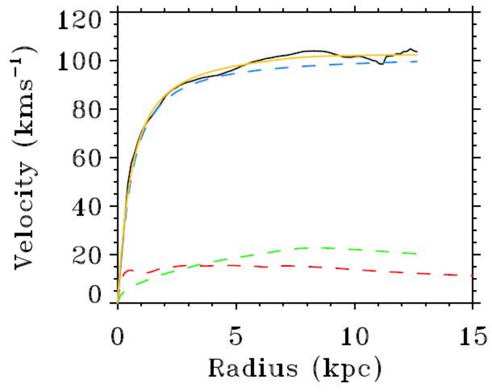
Black: Observed rotation curve

Green: Rotation due to gas

Red: Rotation due to stars

Blue: Rotation due to dark halo

Yellow: Total rotation curve



The disk M/L is not yet constrained by the flaring!

Now look at the gradient of the vertical force $\partial K_{\rm z}/\partial z$ as a function of R, deduced from the flaring and HI velocity dispersion at those radii using

$$\frac{\partial}{\partial z}\left[\sigma_{\rm g}^2(R)\ln\rho_{\rm g}(R,z)\right] = -\frac{\partial K_{\rm z}(R,z)}{\partial z}$$

For Gaussian gas density distributions this gives

$$\frac{\partial K_{\rm z,tot}(R,z)}{\partial z} = \frac{\sigma_{\rm g}^2(R)}{(\mathrm{FWHM}_{\rm g}(R)/2.355)^2}$$

and is constant in z.

We use for this gradient the dark halo from the equations for the flattened pseudo-isothermal halo.

For the stars and gas this gradient can be determined using Poisson's equation

$$\frac{\partial K_{\mathrm{z,i}}(R,z)}{\partial z} = -4\pi G \rho_{\mathrm{i}}(R,z) + \frac{1}{R}\frac{\partial (R K_{\mathrm{R,i}})}{\partial R}$$

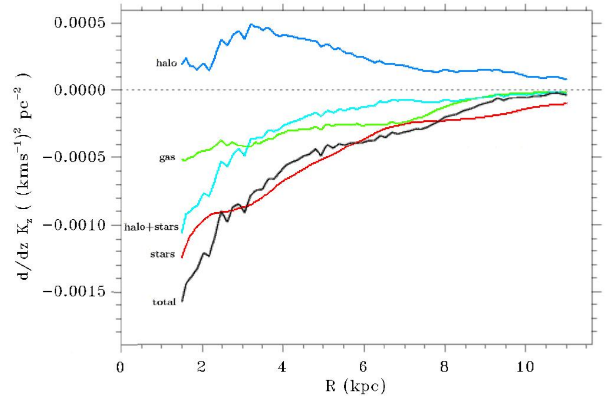
In terms of rotation velocities, due to these components

$$\frac{\partial K_{\mathrm{z,i}}(R,z)}{\partial z} = -4\pi G \rho_{\mathrm{i}}(R,z) + \frac{1}{R}\frac{\partial V^2_{\mathrm{rot,i}}(R)}{\partial R}.$$

Then we determine the gradient due to the dark matter halo

$$\frac{\partial K_{\mathrm{z,halo}}(R,z)}{\partial z} = \frac{\partial K_{\mathrm{z,tot}}(R,z)}{\partial z} - \frac{\partial K_{\mathrm{z,stars}}(R,z)}{\partial z} - \frac{\partial K_{\mathrm{z,gas}}(R,z)}{\partial z}$$

First for the decomposition of the rotation curve just shown.



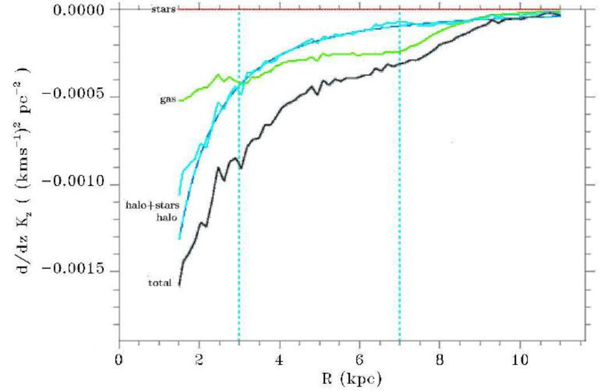
Obviously this disk M/L overestimates the disk mass.

Models were then fit the curve 'halo+stars' with three free parameters: the asymptotic rotation of the halo, the stellar M/L and the flattening q.[34]

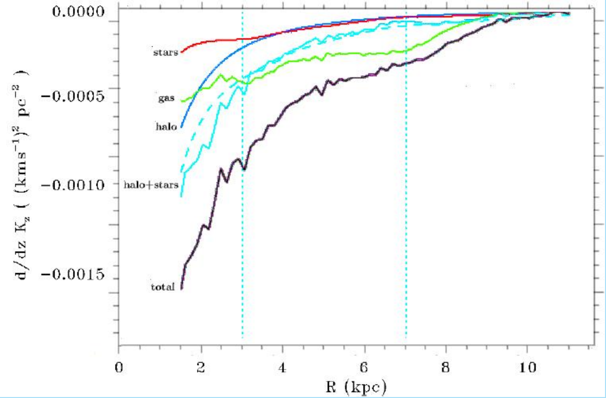
This produced with least-squares minimisation a best shape of $q = 1.0 \pm 0.1$, independent of the other parameters.

The best fit actually occurs for $M/L = 0$, but $M/L = 0.2$ is almost as good.

[34] Remember that the core radius of the halo was fixed by the rotation curve analysis.



The actual fits were done over the range $R = 3$ to 7 kpc, but were successful over $R = 2$ to 9 kpc.



This is the fit for $M/L = 0.2$.

Conclusions

- Galaxies with high rotation velocity are larger and thicker and have higher stellar velocity dispersions.
- Disks are in general not maximal.
- Very high surface brightness disks may be maximal.
- Low surface brightness disk are more flattened and have smaller stellar velocity dispersions.
- Maximal disks appear to have more anisotropic velocity dispersions or have larger values of Q.
- HI layer flaring can also be used to constrain the flattening of the dark matter halos.
- In NGC 4244 the halo appears highy flattened, but in UGC 7321 is seems almost spherical.
- Other galaxies in O'Brien's sample remain to be analysed.