Abundances and Kinematics from High-Resolution Spectroscopic Surveys

Lecture 2: Formation of a Stellar Spectrum

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I have a spectrum: what next?

Bruno Letarte 2007, PhD thesis

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Spectral Types

Astronomers in the 19th Century classified stars according the strength of the Balmer lines of neutral hydrogen, with A stars having the strongest lines, B stars next strongest, etc. Many sub-classes subsequently fell into disuse.

In the 1880s Antonia Maury (Harvard) realised that when classes were arranged in the order O B A F G K M the strength of ALL the spectral lines (not just H), changed continuously along the sequence.

Between 1911 and 1949 almost 40 000 stars were classified into the HENRY DRAPER CATALOGUE and its supplements.

We now know that Maury’s spectral sequence lists stars in order of decreasing surface temperature, and each class has been sub divided into subclasses, from 0, hottest to 9, coolest. Our Sun is a G2 star, for example.
Spectral Types: temperature sequence

- **T ~ 4000K**
  - Molecules!
  - Mainly neutral metal lines

- **T ~ 6000K**
  - Ionised Metal lines
  - T < 11 000K
  - Dominated by neutral H

- **T ~ 30 000K**
  - Highly ionised species

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I selected my source from a CMD

D = 79 kpc
$r_{\text{core}} = 5.8'$

130 candidates
Part 2a: A Model Stellar Atmosphere

Stellar atmospheres are the connecting link between observations and the rest of stellar astrophysics: they are interesting for their own sake but I am only going to discuss them as a tool to understand stellar abundances - to try to understand galaxy formation and evolution processes.

**Textbooks:**

*The Observation & Analysis of Stellar Photospheres* by D.F. Gray (CUP)

*An Introduction to Modern Astrophysics* by B.W. Carroll & D.A. Ostlie

**Materials on the Web:**


??, by Dr. J.B. Tatum, UVic
Why do we need a Model Atmosphere?

A stellar spectrum is not simply related to the physical state of the atmosphere: there are many physical variables that require carefully modeling.

An initial model can be constructed on the basis of observations and known physical laws. This model is then modified and improved iteratively, and when the model closely matches all the available observations we may feel that we can begin to deduce certain properties of a star:

- temperature, surface gravity, radius, chemical composition, rate of rotation, etc as well as the thermodynamic properties of the atmosphere itself.

These are basic models, but indispensable for analysis of stellar spectra, usual simplifications:

- **Plane-parallel geometry**, making all physical variables a function of only one space coordinate

- **Hydrostatic Equilibrium**, no large scale accelerations in photosphere, comparable to surface gravity, no dynamical significant mass loss.

- **No fine structures**, such as granulation, starspots

- **Magnetic fields are excluded**
The Hydrostatic Equation

The difference in pressure between the top and bottom of a volume. The relation between pressure and optical depth is established from the assumption of Hydrostatic Equilibrium.

\[ \frac{dP}{d\tau_v} = \frac{g}{\kappa_v} \]

In most stars the bulk of the total pressure is gas pressure. To integrate the hydrostatic equation we need to understand the temperature and pressure gradients in a stellar atmosphere:

Limb darkening exists because the continuum source function decreases outwards, as we look toward the limb we see systematically higher photospheric layers which are less bright.
Stellar Photosphere

The geometrical extent of the photosphere in stars differs inversely with the surface gravity

\[ g = \frac{g_\odot M}{R^2} \]

...and the opacity of the gas in the photosphere

Temperature affects the nature of the atmosphere, because this varies strongly in photosphere - need to define an "effective" temperature

The major portion of visible stellar spectrum originates in photosphere
Stars can be treated as Black bodies...

The basic condition for the black body as an emitting source is that a negligible small fraction of the radiation escapes.

At the bottom of the stellar photosphere the optical depth to the surface is high enough to prevent escape of most photons. They are reabsorbed close to where they were emitted - thermodynamic equilibrium - and radiation laws of BB apply.

However, a star cannot be in perfect thermodynamic equilibrium - there is a net outflow of energy (however defined) that varies with position - the distribution of particles represents the range of temperatures - local thermodynamic equilibrium

Higher layers deviate increasingly from Black Body case as this leakage becomes more significant. There is a continuous transition from near perfect local thermodynamic equilibrium (LTE) deep in the photosphere to complete non-equilibrium (non-LTE) high in the atmosphere.
(Local) Thermodynamic Equilibrium

Thermodynamic Equilibrium is applied to relatively small volumes of the model photosphere - volumes with dimensions of order unity in optical depth. For this reason the name **LOCAL THERMODYNAMIC EQUILIBRIUM** (LTE) is used.

The photosphere may be characterized by one physical temperature at each depth.

(L)TE means that the components of a gas (atoms, electrons, photons) interact enough that the energy is distributed equally among all possible forms (kinetic, radiant, excitation etc), and the following theories can be used to understand physical processes:

- Distribution of photon energies: Planck Law (Black-Body Relation)
- Distribution of kinetic energies: Maxwell-Boltzmann Relation
- Distribution among excitation levels: Boltzmann Equation
- Distribution among ionization states: Saha Equation

Together these equilibrium equations account for the basic features of stellar spectra
Stellar Temperature

Different Ways to determine:

• Effective Temperature (Stefan-Boltzmann relation)
• Excitation Temperature (Boltzmann Eqn)
• Ionization Temperature (Saha Eqn)
• Kinetic Temperature (Maxwell Boltzmann Distribution)
• Colour Temperature (Wien’s Law)

These temperatures will be the same for a simple gas of gas confined in a box, i.e. In Thermodynamic Equilibrium
Temperature derived from Planck

From the Stefan-Boltzmann relation, the effective temperature ($T_{\text{eff}}$) of a star is:

$$L = 4\pi R^2 \sigma T_{\text{eff}}^4$$

Fitting the shape of the star’s continuous spectrum to the Planck function (Wien’s law), gives the Colour-Temperature of a star:

$$\lambda_{\text{max}} T = \text{constant}$$
Boltzmann Equation

The Boltzmann equation describes the distribution of atoms of a particular ionization state among their various possible excited levels at a given temperature, \( T \)

\[
\frac{N_b}{N_a} = \frac{g_b}{g_a} e^{-\frac{(E_b - E_a)}{kT}}
\]

\( N \), number of atoms; \( g \), statistical weight; \( a \) & \( b \), represent 2 different excitation levels, with the energy of \( b \) higher than the energy of \( a \)

The relative population of each level depends in a detailed way upon the mechanisms for populating and de-populating them: radiative, collisional & spontaneous - when collisions dominate - LTE

Can determine the “Excitation Temperature” of a star if LTE applies.
Maxwell-Boltzmann Law

When particles interact sufficiently they spread the available energy around and an equilibrium distribution evolves which means that the velocities of particles in thermodynamic equilibrium in a hot gas are distributed with a Gaussian or Maxwellian velocity distribution.

\[ p(v)dv = \left( \frac{m}{2\pi kT} \right)^{\frac{3}{2}} e^{-\frac{mv^2}{2kT}} \frac{4\pi v^2}{4\pi} dv \]
Saha Equation (Ionization Temperature)

The Boltzmann equation can be adapted to include states above the ionization potential of the atom. This gives the ratio of atoms in two different ionization states (e.g. ratio of HI to HII).

The calculation is quantum in nature and involves the Planck constant. For an ideal gas (collision dominated):

$$\frac{N_{i+1} P_e}{N_i} = \frac{2kT Z_{i+1}}{Z_i} \left( \frac{2\pi m_e kT}{\hbar^2} \right)^\frac{3}{2} e^{-\chi_i/kT}$$

$P_e$ drives the equilibrium towards neutral state when the electron density (pressure) is high and towards the ionized state when it is low.

i.e., electron pressure in a stellar atmosphere changes the line strengths
The Temperature of a star

These temperatures are the same for the ideal case of a gas confined within a box, i.e. a gas in equilibrium - has a single well defined temperature.

Figure 9.5 The spectrum of the Sun. The dashed line is the curve of an ideal blackbody having the Sun's effective temperature. (Figure from Aller, Atoms, Stars, and Nebulae, Revised Edition, Harvard University Press, Cambridge, MA, 1971.)
Opacity:

As a precursor to calculating the transfer of radiation through a model stellar photosphere need to first consider the continuous absorption coefficient, or opacity. The detailed calculation of $\kappa_\nu$, the opacity, is a tough and extended problem within astrophysics, and cross-sections as a function of frequency for all sorts of atoms and molecules and even dust grains have to be calculated.

The wavelength dependence shapes the continuous spectrum emitted by a star. It also has a major influence on the temperature structure of an atmosphere because the opacity controls how easily energy flows at a given wavelength.

Where opacity is high, energy flow (flux) is low.
What is Opacity?

Any process which removes photons from a beam of parallel light travelling through a gas will be called absorption, which includes scattering

\[ dI_\lambda = -\kappa_\lambda \rho I_\lambda ds \]

The change in intensity, \( dI_\lambda \), of a ray of wavelength, \( \lambda \), as it travels through a gas is proportional to its intensity, \( I_\lambda \), and the distance travelled, \( ds \), and the density of the gas, \( \rho \). \( \kappa_\lambda \) is the absorption coefficient, or opacity.

**Opacity is a function of composition, density & temperature.**

It is determined by the details of how photons interact with particles (atoms, ions, free electrons).

If a photon passes within \( \sigma_\lambda \) of a particle (cross-section) the photon may be either absorbed or scattered - both remove the photon from the beam of light, and so contribute to the opacity, \( \kappa_\lambda \), of the material.

If the opacity varies slowly with \( \lambda \) it determines the stars continuous spectrum (or continuum). The dark absorption lines superimposed on the spectrum are the result of a rapid variation of opacity with \( \lambda \).
Sources of Opacity

In general there are 4 primary sources of opacity - each involves a change in the quantum state of an electron.

**Bound-Bound absorption, \( \kappa_{\lambda,bb} \)**

Small, except at those discrete wavelengths capable of producing a transition. i.e., responsible for forming absorption lines

**Bound-Free absorption, \( \kappa_{\lambda,bf} \)**

Photoionisation - occurs when photon has sufficient energy to ionize atom. The freed e\(^-\) can have any energy, thus this is a source of continuum opacity.

**Free-Free absorption, \( \kappa_{\lambda,ff} \)**

A scattering process. A free electron absorbs a photon, causing the speed of the electron to increase. Can occur for a range of \( \lambda \), so it is a source of continuum opacity.

**Electron scattering**

A photon is scattered, but not absorbed by a free electron. A very inefficient scattering process only really important at high temperatures - where it dominates
Opacity & Stellar type

The primary source of opacity in most stellar atmospheres is: the photoionization of H\(^+\) ions, but these become increasingly ionized for A, B stars, and then photoionization of H atoms and free-free absorption become the main sources.

For O stars the main source is electron scattering, and the photoionisation of He also contributes.

Molecules can survive in cooler stellar atmospheres and contribute to bound-bound and bound-free opacities. The large numbers of molecular lines are an efficient impediment to the flow of photons.
Radiative Transfer

The transfer equation is the basic tool that describes the passage of light through a star’s atmosphere:

\[- \frac{1}{\kappa_\lambda \rho} \frac{dI_\lambda}{ds} = I_\lambda - S_\lambda\]

In an equilibrium (or steady-state) star there can be no change in the total energy contained within any layer of the stellar atmosphere. This means that mechanisms involving absorbing and emitting must be in balance throughout the star.

It is a troublesome equation. The intensity of light depends on the direction of travel to account for the net outflow of energy; although absorption and emission coefficients are the same for light traveling in all directions. They depend upon T and ρ in a rather complicated way.

To know anything about physical conditions in a stellar atmosphere (T, ρ) we must know at what depth the spectral lines are formed, which means understanding and solving the radiative transfer equation.

Powerful simplification: PLANE-PARALLEL ATMOSPHERE

i.e., atmospheric curvature >> thickness of the atmosphere
Sources of Continuum Opacity

- Stimulated emission factor (not always included)
- Neutral hydrogen (bound-free & free-free absorption)
- Hydrogen molecules (plentiful in cool stars)
- Helium (free-free He absorption, can be significant at long λ in cool stars)
- The metals (C, Si, Al, Mg & Fe), bound-free opacity, important in UV
- Scattering, by free electrons, same efficiency at all λs
- Line Opacity, cumulative effect of many lines; depends on T, P, Z & v_t
A Stellar Atmosphere model

e.g., MARCS models (Gustafsson et al. 1975 A&A, 42, 407) computes plane-parallel, line blanketed, flux constant stellar atmospheres.

Specify: $T_{\text{eff}}$, $\log_{10} g$ & $Z$ (metallicity)

Output: optical depth, $\tau$; temperature, $T$; log pressure (gas); log pressure ($e^-$); mean molecular weight, $\mu$; opacity, $\kappa$

This “atmosphere” (temperature-pressure relation) is then fed into a spectral synthesis programme (e.g., MOOG, Sneden 1973 or CALRAI, Spite 1967), and the abundances required to fit each observed line will be calculated and returned.
Part 2b: Stellar Absorption Line formation

A cool, thin gas seen in front of a hot source produces absorption lines: in the continuum region, $\tau$ is low and we see primarily the background source. At the wavelengths of spectral lines, $\tau$ is large and we see the intensity characteristic of the temperature of the cool gas. Since this is lower than central source, these appear as absorption features.
Line profiles

A typical stellar spectrum, especially for cooler stars, contains thousands of lines, from many different chemical elements. Spectral lines have a variety of widths for which there are several causes, some internal to the atom, others external, and each produces a characteristic profile. Some types of profile have a broad core and small wings; others have a narrow core and extensive, broad wings.

We can study:

• types of nuclear reactions; internal mixing of material; penetration depths of convection zones; diffusion and gravitational settling; accretion.
• Li & Be abundances give information on the age of the star.
• Gross differences in the chemical make up of the star may affect the strength of convection and consequently any magnetic activity stemming from a dynamo.
• The evolution of a galaxy may be traceable in part through differences in chemical abundances.
Structure of Spectral Lines

Line Widths have three main components:

- Natural broadening (Lorentzian profile, very narrow)
- Doppler broadening (Maxwell-Boltzmann distribution, gaussian)
- Pressure (collisional) broadening (Lorentzian profile)

**Voigt profile: doppler + pressure (damping) profiles**

Doppler dominates the centre, and damping dominates the wings, especially for broad lines.

The simplest model for calculating the line profile assumes that the star’s photosphere acts as a source of black-body radiation and the atoms above the photosphere remove photons from this continuous spectrum to form absorption lines.

\( T, \rho, Z \) must be adopted for the line formation region (above photosphere)

\( T, \rho \) determine the importance of doppler and pressure broadening, and are also used in the Boltzmann and Saha equations. The calculation of a spectral line depends not only on the abundance of the element forming the line but also on quantum mechanical details of how atoms absorb photons.
Natural Line Width

The bound-bound absorption problem is analogous to the mechanical system of a damped, driven harmonic oscillator. It is of course a quantum problem, the form of the solution and the terminology employed, echo its classical roots.

The “damping constant” in the classical mechanical analog it is related to the viscosity of the liquid which is causing the damping. In the classical electrodynamic case, the damping is the induced radiation of accelerated electrons, so this is called radiation damping, and it sets the Lorentzian profile and natural width of a line, which is intrinsically very small, but the wings can be important for strong lines.
Pressure Broadening

Spectral lines can be broader in higher pressure atmospheres for a variety of reasons. Together, these are referred to as “pressure broadening”, and produces a Lorentzian profile.

**Stark Effect**: the splitting of lines in a constantly fluctuating external electric field generated by nearby ions and electrons. The net effect, is to broaden the line.

**Collisional de-excitation**: Anything which lowers the lifetime of an atom in an excited state will also broaden the spectral lines associated with transitions to or from that state. Collisional de-excitation in which an atom collides with an electron or other particle and the excitation energy is transferred to the colliding particle without the generation of a photon is an example. Obviously, when the pressure in an atmosphere is higher, the influence of collisions on the spectral line will be largest: white dwarfs are the most extreme examples of pressure broadening.
Doppler Broadening

Spectral lines in stars have widths that are much larger than the natural prediction predominantly because the random velocities of the atoms spread the natural frequencies over a range by the Doppler effect.

For temperatures typical of stellar atmospheres, this greatly exceeds natural broadening. This effect dominates in most stars and most lines and produces a Gaussian shape (being the Maxwell-Boltzmann distribution). Because the Gaussian profile is an exponential, it does fall off very rapidly in the wings, and because of that, the Lorentzian wings are important for strong lines.

The combined Lorentzian and Gaussian shapes are known as the Voigt profile and are representative of most real spectral lines:
Zeeman Splitting

In very strong magnetic fields atoms can align in quantum ways causing slight separations in the energies of atoms in the same excitation levels. This “splits” the lines into multiple components. The stronger the field, the greater the splitting. This is a way to measure magnetic field strengths of stars. Often the splitting cannot be resolved but effectively broadens the line.
Hyperfine structure

A small perturbation in the energy levels of an atom due to the interaction of the nuclear magnetic dipole with the magnetic field of the electron. This phenomenon is energy level dependent, and therefore different for each line.

It is not often evident in the visible spectrum of stars. Generally the resolution is too poor and the lines are so broadened by high temperature as to mask any hyperfine structure. However in some cases the hyperfine structure of the lines, even if not fully resolved, is sufficient to make the lines noticeably broad, e.g., Eu

**Figure 4.18:** HFS correction (dex) for the Eu line at 6645.1Å, tested on a plane-parallel model of $T_{\text{eff}} = 4300$, $\log g = 0.6$, $[\text{Fe/H}] = -1.5$ and $v_t = 1.7$. 

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Part 2c: Some Examples
Hydrogen Lines

In the visible part of the spectrum these are Balmer transitions which are from the n=2 excited level. They are strongest in A stars because that is where N (HI, n=2) is highest. Hotter stars have more HII than HI, while cooler stars have more HI in the ground state (n=1).

Stark Effect
Helium Lines

Helium is the second most abundant element, but it is not seen in most stars because the ground state of its neutral atom (HeI) is very stable. It is a “full shell” which makes it a noble gas and it does not participate much in chemical reactions. Only in the very hottest stars (O and B) do He atoms show up in their excited levels where they can absorb visible light. For the very hottest O stars we also see HeII lines.
Metal Lines

Strongest when temperature is low enough that lower ionization stages are populated.

The strengths of lines depend sensitively on the level population of the atoms. If there is a transition from a ground state to a first excited state in the wavelength range observed then an atom will have a deep absorption line in the spectrum (e.g. NaI), a “resonance line”.

The metal lines become progressively stronger as the temperature cools and dominate in the F, G, K stars.
Molecular Bands

For very cool stars (M, L, T type) the atmospheres are sufficiently cool that simple molecules can form. These can absorb not only in electronic transitions, but also in vibrational and rotational modes. These create “bands” of absorption which can reduce the flux in vast portions of the spectrum. In M stars, TiO is a common important molecule. In L and T stars, other molecules such as CO, H$_2$O and CH$_4$ become important.
Relative Strength of Spectral Lines
Dominant Features in the Spectra of Stars

Temperature (K)

50,000  25,000  10,000  8000  6000  5000  4000  3000

Line strength

He II  He I  Mg II  Si III  Si II  Ca II  Fe II  Fe I  Ca I  TiO

Spectral type

O5  B0  A0  F0  G0  K0  M0  M7
Summary

The Boltzmann and Saha equations explain the primary features of stellar spectra and the Planck law explains the overall continuum shape.