Fourth set of problems for the course on Galaxies, 2005-2006

1. Elliptical galaxies

   (a) If the luminosity density of stars in a galaxy is $j(r) = j_0(r_0/r)^\alpha$, show that the surface brightness at distance R from the center is

   $$I(R) = I_0(r_0/R)^{\alpha-1} \tag{1}$$

   as long as $\alpha > 1$. What happens if $\alpha < 1$? Compute the total luminosity of the system.

   (b) The fraction of galaxies with apparent axis ratios $(q_0, q_0 + dq_0)$ observed from a random direction θ and true axis ratios $q = \beta/\alpha$, is given by

   $$f(q_0)dq_0 = \frac{q_0 dq_0}{\sqrt{(1-q^2)(q_0^2-q^2)}}. \tag{2}$$

   Show that if we view these galaxies from random directions, the fraction of oblate elliptical galaxies with true axis ratio q that appear more flattened than q_0 is

   $$F(< q_0) = \int_q^{q_0} f(q_0')dq_0' = \sqrt{\frac{q_0^2 - q^2}{1-q^2}} \tag{3}$$

   If these galaxies have $q = 0.8$, show that the number seen in the range $0.95 < q_0 < 1$ should be about one-third of those with $0.8 < q_0 < 0.85$. Show that for smaller values of q, an even higher proportion of the images are nearly circular, with $0.95 < q_0 < 1$. You can do the same exercise for $q = 0.6$, and compare your results to the figure shown in class, and explain why it is unlikely that all these galaxies have oblate shapes.

   (c) The virial theorem relates the internal potential energy W and the kinetic energy K of a system in equilibrium through: $2K + W = 0$. Assuming that both the velocity dispersion σ and the mass-to-light ratio M/L are constant throughout a galaxy, and that no dark matter is present, use the virial theorem to show that

   - Since the potential energy $W \propto -GM^2/R_e$, where M is the total mass of the galaxy and R_e its effective radius, and the kinetic energy $K \sim M\sigma^2/2$, so the mass of the galaxy should be $M \propto \sigma^2 R_e$.
   - If the surface brightness $I(R)$ of all elliptical galaxies could be described by Sersic's law $I(R) = I_e \exp[-b(R/R_e)^{1/n} - 1]$ with the same value of n, explain why their total luminosity L should follow $L \propto I_e R_e^2$.
   - If all elliptical galaxies had the same mass-to-light ratio M/L and surface brightness at the effective radius I_e, the Faber-Jackson relation is expected.

2. Disk galaxies

   In a galaxy where the potential follows the Plummer model

   $$\Phi(r) = \frac{-GM}{\sqrt{r^2 + a^2}} \tag{4}$$

   find the rotation curve $V(r)$. Show that $V_{max}^2 = 2GM/(3\sqrt{3}a)$. Sketch $V(r)$ for $r \leq 4a$. For inclination $i = 30^o$, draw a spider diagram with contours at 0.2, 0.4, 0.6, and 0.8 of V_{max}.