Third set of problems for the course on Galaxies, 2005-2006

1. Chemical evolution

   In this problem, we will investigate a more elaborate version of the accretion model discussed in class. We assume that initially all of the disk mass is in metal-free gas, and that a constant fraction $(1-q)$ of each infalling gas parcel δM_t is locked up by star formation. Thus the corresponding change in gas mass is $\delta M_g = q\delta M_t$.

   (a) Using that

   $$\frac{dZ}{dM_t} = \frac{1}{M_g}\left[p - Z - p\frac{dM_g}{dM_t}\right] \tag{1}$$

   show that a parametric solution for $Z(M_g)$ is $Z = p(1-q)(1-e^{-u})$, and $M_g = M_g(0)e^{qu}$, where $M_g(0)$ is the initial gas mass.

   (b) Show that the ratio of the stellar mass at t_1 to the mass in gas at the present time t_0 is

   $$\frac{M_s(u_1)}{M_g(u_0)} = \frac{1-q}{q}(e^{qu_1}-1)e^{-qu_0} \tag{2}$$

   where u_i is the value of the parameter u at time t_i.

   (c) Now consider the case $u_0 >> 1/q$, and let u_1 be an epoch at which the metallicity Z_1 was substantially lower than Z_0. Show that (i) the present metallicity is $Z_0 \sim p(1-q)$; (ii) that $u_1 \sim -\ln(1 - Z_1/Z_0) << 1$; (iii) that

   $$\frac{M_s(u_1)}{M_g(u_0)} \sim -\ln\left(1 - \frac{Z_1}{Z_0}\right)e^{-qu_0} \tag{3}$$

   This formula differs from that obtained for the simple accretion model only by the presence of the factor e^{-qu_0}. Since this factor can take any value between 0 and 1 depending on q and u_0, it follows that in this modified accretion model, the fraction of low metallicity stars can be made arbitrarily small.

2. Dynamics.

   (a) In a spherical galaxy, the gravitational potential $\Phi(r)$ is

   $$\Phi(r) = -\frac{GM(<r)}{r} - 4\pi G\int_r^\infty \rho(r')r'dr' \tag{4}$$

   Check that by differentiating this expression with respect to r you recover Newton's second theorem.

   (b) Show that at radius r inside a uniform sphere of density ρ, the radial force $F_r = -4\pi G\rho r/3$. If the density is zero for $r > a$, show that

   $$\Phi(r) = -2\pi G\rho(a^2 - \frac{r^2}{3}), \tag{5}$$

(c) The collision-timescale is

$$t_{coll} \sim \frac{V^3}{4\pi G^2 m^2 n} \tag{6}$$

where n is the number density of particles in a system, m is their mass, and V is their typical (relative) speed, i.e $V^2 = GNm/R$ where R is a characteristic scale of the system, and N the total number of particles (for a spherical object $N = 4\pi n R^3/3$). The relaxation timescale is

$$t_{relax} \sim \frac{N}{8 \log \Lambda} t_{cross} \tag{7}$$

where $\Lambda = R/R_i$ with R_i the characteristic size of the particles, and $t_{cross} = R/V$ the crossing time. Compute t_{coll} and t_{relax} for a globular cluster, an elliptical galaxy and for a cluster of galaxies using the parameters below. What do you conclude from the comparison? Which process are important for stars in a cluster? Which for galaxies in a cluster?

object	N	R_i	m	R
globular cluster	10^5	7×10^5 km	1 M_{sun}	4 pc
elliptical galaxy	10^{11}	7×10^5 km	1 M_{sun}	10 kpc
galaxy cluster	10^3	100 kpc	$10^{12} M_{sun}$	3 Mpc