Second set of problems for the course on Galaxies, 2005-2006

1. Spatial distribution of stars in the Galaxy

   (a) The density $n_S(R, z, \phi)$ of stars of a particular type S in the disk can be approximated by a double exponential:

   $$n_S(R, z, \phi) = n_S(0) \exp[-R/h_R(S)] \exp[-|z|/h_z(S)] \tag{1}$$

   where h_R is the scale length, and h_z is the scale height of the disk, and which may vary with S type. Integrating the above equation, show that at radius R the surface density of stars of type S is $\Sigma_S(R) = 2n_S(0)h_z \exp[-R/h_R]$. If each star has luminosity L_S, the surface brightness is $I_S(R) = L_S\Sigma_S(R)$. Assuming that the scale height and scale lengths are independent of S, show that the disk's total luminosity is $L_D = 2\pi I(0)h_R^2$. For the Milky Way, taking $L_D = 1.5 \times 10^{10} L_\odot$ in the V-band, and $h_R = 3$ kpc, show that the disk's surface brightness at the position of the Sun (8 kpc from the Galactic center) is $\approx 18L_\odot/\text{pc}^2$. Since the mass density in the disk is $40-60M_\odot/\text{pc}^2$, the $M/L_V \sim 2-3$. Why is this larger than M/L_V for stars within 100 pc from the Sun?

   (b) Here we make a model describing the distribution of stars and the way we observe them, to explore the Malmquist bias.

   - Your model sky consists of G-type stars in regions A (85 pc < d < 95 pc), B (95 pc < d < 105 pc) and C (105 pc < d < 115 pc). If the density is uniform, and you have 10 stars in region B, how many are there in regions A and C (round to the nearest integer)?.
   - G stars do not all have exactly the same luminosity; if the variation corresponds to about 0.3 magnitudes, what fractional change in luminosity is this? For each of the stars in a given region, roll a die, note the number N on the upturned face, and give your star $M_V = M_{V,\odot} + 0.3 * (N - 3.5)$. If you like to program, you can use more stars, place them randomly in space, and choose the absolute magnitudes from a Gaussian random distribution with mean $M_{V,\odot}$ and variance 0.3.
   - To "observe" your sky, use a "telescope" that can see only stars brighter than apparent magnitude $m_V = 9.8$; these stars are your sample. Assume for simplicity that all stars in region A are at d=90 pc, in B at 100 pc and in C at 110 pc. How different is their mean absolute magnitude from that for all the stars that you placed on the sky?

2. Kinematics of stars and gas in the Galaxy

   (a) In the construction of a catalog of nearby stars, the "solar neighborhood" was defined to be a sphere of 40 pc in diameter. By how much does the standard of rest vary across this sphere? Use the following values for the Oort constants $A = 14$ km/s kpc^{-1} and $B = -12$ km/s kpc^{-1}.

   (b) For a simple model of the Galaxy with $V(R) = 220$ km/s everywhere, find the line of sight velocity as a function of Galactic longitude $V_{los}(l)$ for gas on circular orbits at R = 4, 6, 10, and 12 kpc. Do this by varying the Galactocentric azimuth ϕ around each ring; find d and l for each (ϕ, R). Make a plot similar to Fig.2 (shown in the last class; Fig.2.18 from Sparke & Gallagher), showing the gas on these rings.

   (c) Show that if the rotation curve is flat, with $V(R) = V_0$, then the Oort constants satisfy $A + B = -dV/dR = 0$, and $A - B = V_0/R_0$. Do the measured values of A and B near the Sun correspond to a rising or a falling roation curve? What effects may cause $A + B \neq 0$ when measured near the Sun, even if the Milky Way's rotation speed is roughly constant?