Luminosity evolution of stellar populations

We want to understand how the luminosity and colours of a stellar population (in this case, a coeval group of stars) evolve in time.

The total luminosity can be split into two contributions

$$L(t) = L_{MS}(t) + L_{GB}(t)$$

from the main sequence and the giant branch respectively.

We will now compute each of these terms separately. Our main assumptions are:

a) the luminosity of a star located on the <u>main sequence</u> is a power-law of the mass

$$L = A \left(\frac{M}{M_\odot}\right)^\alpha L_\odot \qquad \alpha \sim 4 \qquad (1a)$$

(we take $A = 1$ for simplicity).

b) the luminosity of a star located on <u>giant branch</u> has a characteristic value L_{giant}

c) the main sequence lifetime τ_{MS} of a star depends on its mass

$$\tau_{MS} = 10\ \mathrm{Gyr} \cdot \left(\frac{M_\odot}{M}\right)^{\gamma} \qquad (1b)$$

(we have set the proportionality such that a $1 M_\odot$ star spends 10 Gyr on the main sequence).

d) The initial mass function is a power-law of mass

$$\xi(M) = \frac{(x-1)}{M_\odot}\left(\frac{M_{low}}{M_\odot}\right)^{x-1} \times \left(\frac{M_\odot}{M}\right)^{1+x} \qquad (1d)$$

where $x > 1$ and $M_{low} \equiv$ the minimum mass formed.

(Note that this expression is simply $\xi(M) = \xi_0\, M^{-(1+x)}$)

We are now ready to compute the luminosity evolution from stars on the main sequence

$$L_{MS}(t) = \int L\, dN = \int_{M_{low}}^{M_{TO}(t)} L(M)\, N_0\, \xi(M)\, dM$$

where $M_{TO}(t)$ is the mass of a star on the turn-off at time t.

Replacing Eq. (1a) and (1d)

$$L_{MS}(t) = L_\odot N_0 (x-1) \left(\frac{M_{low}}{M_\odot}\right)^{x-1} * M_\odot^{x-\alpha} \int_{M_{low}}^{M_{TO}(t)} M^{\alpha} M^{-(1+x)} dM$$

$$\rightarrow \boxed{L_{MS}(t) = L_\odot N_0 \frac{(x-1)}{(\alpha-x)} \left(\frac{M_{low}}{M_\odot}\right)^{x-1} \left(\frac{M_{TO}(t)}{M_\odot}\right)^{\alpha-x}} \quad (2)$$

we have used here that

i) $\alpha - x > 0$

ii) $M_{TO}^{\alpha-x}(t) >> M_{low}^{\alpha-x}$

because $M_{low} \lesssim 0.1\, M_\odot$

and $M_{TO} \sim 1\, M_\odot$ for $t \approx$ Hubble time

Note that this equation implies that the luminosity due to main sequence stars is larger the younger the age of the population (since this is when the mass of a star on the T.O is highest.)

We go on to compute the luminosity evolution from stars located on the giant branch

$$L_{GB}(t) = \int L\, dN = L_g\, N_{GB}(t)$$

where we have used hypothesis (1c), and $N_{GB}(t)$ is the number of stars on the giant branch at time t. This can be computed as follows. $N_{GB}(t)$ is given by the initial mass function for masses greater than the mass of the T.O

$$N_{GB}(t) = \int_{M_{TO}(t)}^{M_{TO}(t)+\Delta M} N_0\, \xi(M)\, dM$$

$$\simeq N_0\, \xi(M_{TO})\, \Delta M$$

Here we have assumed that the giants all have a similar mass (i.e ΔM is not very large). This is justified by the fact that the giant lifetime τ_{GB} is rather short. Therefore, for a bunch of stars to coincide on the giant branch at a given time, their masses must be very similar.

We can estimate ΔM as follows:

$$\Delta M \sim \left| \frac{dM}{d\tau}\Big|_{TO} \right| \tau_{GB}$$

i.e. the rate of mass going into the GB $\times$
the time since they left the main sequence

Using Eq. (1b) $\rightarrow$ $\left|\frac{dM}{d\tau}\right|_{TO} = \frac{1}{\gamma} \frac{M_{TO}^{\gamma+1}(t)}{M_\odot^{\gamma}\ 10^6 yr}$

Replacing, we get $N_{GB}(t)$ and hence $L_{GB}(t)$ as:

$$\boxed{L_{GB}(t) = N_0 \, L_g \, \frac{x-1}{\gamma} \, \frac{\tau_{GB}}{10\,Gyr} \left(\frac{M_{low}}{M_\odot}\right)^{x-1} \left(\frac{M_{TO}(t)}{M_\odot}\right)^{\gamma-x}} \quad (3)$$

Note that this also depends on the mass of the T.O. at the time of observation. Note as well that $\gamma - x > 0$ (typically $\gamma \sim 3$)

The relative importance of main sequence and giant branch stars for the luminosity budget

This is given by the ratio $L_{MS}(t) / L_{GB}(t)$

which can be expressed as a function of mass of the T.O.

$$\frac{L_{MS}(t)}{L_{GB}(t)} = \frac{\gamma}{\alpha - x} \, \frac{10\,Gyr}{\tau_{GB}} \left(\frac{M_{TO}(t)}{M_\odot}\right)^{\alpha-\gamma} \frac{L_\odot}{L_g}$$

or as function of time

$$\boxed{\frac{L_{MS}(t)}{L_{GB}(t)} = \frac{\gamma}{\alpha - x} \, \frac{10\,Gyr}{\tau_{GB}} \, \frac{L_\odot}{L_g} \left[\frac{10\,Gyr}{t}\right]^{\alpha/\gamma - 1}}$$

This equation explicitly shows that the contrib. of main sequence stars wrt giant branch stars decreases in time.

Typically α and L_g are functions of the passbands

Some values are

$$\alpha_U = 4.9 \qquad L_{gU} = 35\, L_\odot$$

$$\alpha_B = 4.5 \qquad L_{gB} = 60\, L_\odot$$

$$\alpha_V = 4.1 \qquad L_{GV} = 90\, L_\odot$$

For example, we can compute the time when both MS * and GB * contribute the same luminosity in the V-band.

Assuming that $x = 1.35$ (Salpeter) and $\tau_{GB} = 0.3$ Gyr (and for $\gamma = 3$), one finds.

$$\tau_{equality} \simeq 0.84 \text{ Gyr}$$

$$M_{TO}(\tau_{eq}) = 2.28\, M_\odot$$